Section 2.4

2.) a.) T
b.) F
c.) F
d.) T
e.) T
f.) T
g.) T
h.) T
i.) T

3.) a.) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{x}{2}+1\right) = 1+1 = 2$,

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3-x) = 3-2 = 1$

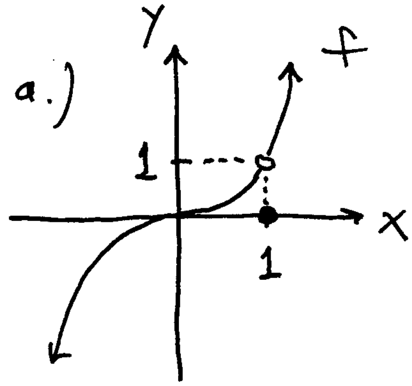
b.) $\lim_{x \to 2} f(x)$ DNE because of a.)

c.) $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \left(\frac{x}{2}+1\right) = 2+1 = 3$,

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \left(\frac{x}{2}+1\right) = 2+1 = 3$

d.) $\lim_{x \to 4} f(x) = 3$ because of c.)

7.) a.)



$$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

b.) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^3 = 1^3 = 1$,

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^3 = 1^3 = 1$

c.) $\lim\limits_{x \to 1} f(x) = 1$ because of b.)

12.) $\lim\limits_{x \to 1^+} \sqrt{\dfrac{x-1}{x+2}} = \sqrt{\dfrac{1-1}{1+3}} = \sqrt{\dfrac{0}{4}} = \sqrt{0} = 0$

15.) $\lim\limits_{h \to 0^+} \dfrac{\sqrt{h^2+4h+5} - \sqrt{5}}{h}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \frac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}}$$

$$= \lim_{h \to 0^+} \frac{(h^2+4h+\cancel{5}) - \cancel{5}}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)}$$

$$= \lim_{h \to 0^+} \frac{\cancel{h}(h+4)}{\cancel{h}\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} = \frac{4}{\sqrt{5}+\sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

17.) b.) $\lim\limits_{x \to -2^-} (x+3) \cdot \dfrac{|x+2|}{x+2}$

$x = -2$

$x+2 < 0$

$$= \lim_{x \to -2^-} (x+3) \cdot \frac{-\cancel{(x+2)}}{\cancel{x+2}}$$

$$= \lim_{x \to -2^-} -(x+3) = -(-2+3) = -1$$

23.) $\lim\limits_{Y \to 0} \dfrac{\sin 3Y}{4Y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{Y \to 0} \left(\dfrac{\sin 3Y}{3Y}\right) \cdot \dfrac{3}{4}$

$$= (1) \cdot \frac{3}{4} = 3/4$$

24.) $\lim\limits_{h\to 0^-} \dfrac{h}{\sin 3h} \overset{"\frac{0}{0}"}{=} \lim\limits_{h\to 0^-} \dfrac{1}{3}\cdot\dfrac{3h}{\sin 3h} = \frac{1}{3}(1) = \frac{1}{3}$

26.) $\lim\limits_{t\to 0} \dfrac{2t}{\tan t} \overset{"\frac{0}{0}"}{=} \lim\limits_{t\to 0} \dfrac{2t}{\frac{\sin t}{\cos t}}$

$= \lim\limits_{t\to 0} 2\cdot\dfrac{t}{\sin t}\cdot\cos t = 2(1)(\cos 0)$

$= 2(1)(1) = 2$

27.) $\lim\limits_{x\to 0} \dfrac{x\csc 2x}{\cos 5x} \overset{"\frac{0}{0}"}{=} \lim\limits_{x\to 0} \dfrac{x}{\cos 5x}\cdot\dfrac{1}{\sin 2x}$

$= \lim\limits_{x\to 0} \dfrac{1}{2}\cdot\dfrac{2x}{\sin 2x}\cdot\dfrac{1}{\cos 5x} = \frac{1}{2}(1)\left(\frac{1}{1}\right) = \frac{1}{2}$

29.) $\lim\limits_{x\to 0} \dfrac{x + x\cos x}{\sin x\cos x} \overset{"\frac{0}{0}"}{=} \lim\limits_{x\to 0} \left(\dfrac{x}{\sin x}\cdot\dfrac{1+\cos x}{\cos x}\right)$

$= (1)\cdot\dfrac{1+1}{1} = 2$

32.) $\lim\limits_{h\to 0} \dfrac{\sin(\sin h)}{\sin h} \overset{"\frac{0}{0}"}{=} \lim\limits_{\substack{k\to 0\\ k=\sin h}} \dfrac{\sin k}{k} = 1$

33.) $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\sin 2\theta} = \lim\limits_{\theta\to 0} \dfrac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta}$

$= \dfrac{1}{2(1)} = \dfrac{1}{2}$

34.) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\sin 4x} = \lim\limits_{x\to 0} \left(\dfrac{5}{4}\cdot\dfrac{\sin 5x}{5x}\cdot\dfrac{4x}{\sin 4x}\right)$

$= \frac{5}{4}\cdot(1)\cdot(1) = \frac{5}{4}$

43.) $-1 \le \sin 2x \le +1 \to \frac{-1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x}$

and $\lim_{x \to \infty} \frac{-1}{x} = \text{"}\frac{-1}{\infty}\text{"} = 0$,

$\lim_{x \to \infty} \frac{1}{x} = \text{"}\frac{1}{\infty}\text{"} = 0$, so by Squeeze

Principle $\lim_{x \to \infty} \frac{\sin 2x}{x} = 0$.

45.) $-1 \le \sin t \le +1 \to \frac{-1}{t} \le \frac{\sin t}{t} \le \frac{1}{t}$

and $\lim_{t \to -\infty} \frac{-1}{t} = \text{"}\frac{-1}{-\infty}\text{"} = 0$,

$\lim_{t \to -\infty} \frac{1}{t} = \text{"}\frac{1}{-\infty}\text{"} = 0$, so by Squeeze

Principle $\lim_{t \to -\infty} \frac{\sin t}{t} = 0$; AND

$-1 \le \cos t \le +1 \to \frac{-1}{t} \le \frac{\cos t}{t} \le \frac{1}{t}$, and

$\lim_{t \to -\infty} \frac{-1}{t} = 0 = \lim_{t \to -\infty} \frac{1}{t}$, so by

Squeeze Principle $\lim_{t \to -\infty} \frac{\cos t}{t} = 0$;

NOW

$$\lim_{t \to -\infty} \frac{2 - t + \sin t}{t + \cos t} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{t \to -\infty} \frac{2 - t + \sin t}{t + \cos t} \cdot \frac{\frac{1}{t}}{\frac{1}{t}}$$

$$= \lim_{t \to -\infty} \frac{\frac{2}{t} - 1 + \frac{\sin t}{t}}{1 + \frac{\cos t}{t}} = \frac{(0) - 1 + (0)}{1 + (0)} = -1$$

47.) $\lim_{x \to \infty} e^{-x} \sin x = \lim_{x \to \infty} \frac{\sin x}{e^x}$;

$-1 \le \sin x \le +1 \to \dfrac{-1}{e^x} \le \dfrac{\sin x}{e^x} \le \dfrac{1}{e^x}$ and

$\lim\limits_{x\to\infty} \dfrac{-1}{e^x} = \text{"}\dfrac{-1}{\infty}\text{"} = 0$, $\lim\limits_{x\to\infty} \dfrac{1}{e^x} = \text{"}\dfrac{1}{\infty}\text{"} = 0$, so

by Squeeze Principle $\lim\limits_{x\to\infty} \dfrac{\sin x}{e^x} = 0$.

49.) $$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to-\infty} \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \cdot \frac{e^x}{e^x}$$

$$= \lim_{x\to-\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{e^{-\infty} - 1}{e^{-\infty} + 1} = \frac{\frac{1}{e^{\infty}} - 1}{\frac{1}{e^{\infty}} + 1}$$

$$= \frac{0 - 1}{0 + 1} = -1$$

54.) $$\lim_{x\to\pm\infty} \frac{3x+7}{x^2-2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\pm\infty} \frac{3x+7}{x^2-2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\pm\infty} \frac{\frac{3}{x} + \frac{7}{x^2}}{1 - \frac{2}{x^2}} = \frac{(0) + (0)}{1 - (0)} = 0$$

58.) $$\lim_{x\to\pm\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\pm\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$$

$$= \lim_{x\to\pm\infty} \frac{9 + 1/x^3}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}} = \frac{9 + (0)}{2 + (0) - (0) + (0)} = \frac{9}{2}$$

63.) $$\lim_{x\to-\infty} \frac{x^{1/3} - x^{1/5}}{x^{1/3} + x^{1/5}} = \text{"}\frac{\infty - \infty}{\infty - \infty}\text{"}$$

$$= \lim_{x\to-\infty} \frac{x^{1/3} - x^{1/5}}{x^{1/3} + x^{1/5}} \cdot \frac{\frac{1}{x^{1/3}}}{\frac{1}{x^{1/3}}} = \lim_{x\to-\infty} \frac{1 - x^{1/5 - 1/3}}{1 + x^{1/5 - 1/3}}$$

$$= \lim_{x \to -\infty} \frac{1 - \frac{1}{x^{2/15}}}{1 + \frac{1}{x^{2/15}}} = \frac{1-(0)}{1+(0)} = 1$$

64.) $\displaystyle \lim_{x \to \infty} \frac{x^{-1} + x^{-4}}{x^{-2} + x^{-3}} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{1}{x^4}}{\frac{1}{x^2} + \frac{1}{x^3}} \cdot \frac{x^2}{x^2}$

$$= \lim_{x \to \infty} \frac{x + \frac{1}{x^2}}{1 + \frac{1}{x}} = \frac{\infty + 0}{1 + 0} = \infty$$

74.) $\displaystyle \lim_{x \to \infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) = \text{"} \infty - \infty \text{"}$

$$= \lim_{x \to \infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) \frac{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}$$

$$= \lim_{x \to \infty} \frac{(\cancel{x^2} + x) - (\cancel{x^2} - x)}{\sqrt{x^2+x} + \sqrt{x^2-x}}$$

$$= \lim_{x \to \infty} \frac{2x}{\sqrt{x^2(1+1/x)} + \sqrt{x^2(1-1/x)}}$$

$$= \lim_{x \to \infty} \frac{2x}{\sqrt{x^2}\sqrt{1+1/x} + \sqrt{x^2}\sqrt{1-1/x}}$$

$$= \lim_{\substack{x \to \infty \\ (x>0)}} \frac{2x}{|x|\sqrt{1+1/x} + |x|\sqrt{1-\frac{1}{x}}}$$

$$= \lim_{x \to \infty} \frac{2x}{x\sqrt{1+\frac{1}{x}} + x\sqrt{1-\frac{1}{x}}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}}$$

$$= \frac{2}{1+1} = \frac{2}{2} = 1 .$$