Section 2.5

1.) $\lim\limits_{x\to 0^+} \dfrac{1}{3x} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

3.) $\lim\limits_{x\to 2^-} \dfrac{3}{x-2} = \text{"}\dfrac{3}{0^-}\text{"} = -\infty$ $\qquad x=2$

6.) $\lim\limits_{x\to -5^-} \dfrac{3x}{2x+10} = \text{"}\dfrac{-15}{0^+}\text{"} = -\infty$ $\qquad x={}^-5$

8.) $\lim\limits_{x\to 0} \dfrac{-1}{x^2(x+1)} = \text{"}\dfrac{-1}{(0^+)(1)}\text{"} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$

13.) $\lim\limits_{x\to \frac{\pi}{2}^-} \tan x = \lim\limits_{x\to \frac{\pi}{2}^-} \dfrac{\sin x}{\cos x}$ $\qquad x=\frac{\pi}{2}$

$= \dfrac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

14.) $\lim\limits_{x\to -\frac{\pi}{2}^+} \sec x = \lim\limits_{x\to -\frac{\pi}{2}^+} \dfrac{1}{\cos x}$ $\qquad x=-\frac{\pi}{2}^+$

$= \dfrac{1}{\cos\frac{\pi}{2}} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

18.) a.) $\lim\limits_{x\to 1^+} \dfrac{x}{x^2-1} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$ $\qquad x=1$

b.) $\lim\limits_{x\to 1^-} \dfrac{x}{x^2-1} = \text{"}\dfrac{1}{0^-}\text{"} = -\infty$

$x=-1$

c.) $\lim\limits_{x\to -1^+} \dfrac{x}{x^2-1} = \text{"}\dfrac{-1}{0^-}\text{"} = +\infty$

d.) $\lim\limits_{x\to -1^-} \dfrac{x}{x^2-1} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$

22.) a.) $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{x^3-4x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 2^+} \dfrac{\cancel{(x-2)}(x-1)}{x\,\cancel{(x-2)}(x+2)}$

$= \dfrac{1}{2\cdot 4} = \dfrac{1}{8}$

b.) $\lim\limits_{x\to -2^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to -2^+} \dfrac{x-1}{x(x+2)}$

$= \text{"}\dfrac{-3}{(-2)(0^+)}\text{"} = \text{"}\dfrac{-3}{0^-}\text{"} = +\infty$ $\qquad x=-2$

c.) $\lim\limits_{x\to 0^-} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 0^-} \dfrac{x-1}{x(x+2)}$

$= \dfrac{-1}{(0^-)(2)} = \text{"}\dfrac{-1}{0^-}\text{"} = +\infty$ $\qquad x=0$

d.) $\lim\limits_{x\to 1^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 1^+} \dfrac{x-1}{x(x+2)}$

$= \dfrac{0}{3} = 0$

e.) $\lim\limits_{x\to 0^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 0^+} \dfrac{x-1}{x(x+2)}$

$= \text{"}\dfrac{-1}{(0^+)(2)}\text{"} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$ $\qquad x=0$

so (because of c.)

$\lim\limits_{x\to 0} \dfrac{x^2-3x+2}{x^3-4x}$ DNE

23.) a.) $\lim\limits_{t\to 0^+} \left(2-\dfrac{3}{t^{1/3}}\right) = 2-\left(\dfrac{3}{0^+}\right) = 2-\infty$

$= -\infty$

b.) $\lim\limits_{t \to 0^-} \left(2 - \frac{3}{t^{1/3}}\right) = 2 - \left(\frac{3}{0^-}\right) = 2 - (-\infty)$

$= 2 + \infty = \infty$

26.) a.) $\lim\limits_{x \to 0^+} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = \text{"}\left(\frac{1}{0^+}\right)\text{"} - \frac{1}{1}$

$= \infty - 1 = \infty$

b.) $\lim\limits_{x \to 0^-} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = \left(\text{"}\frac{1}{0^-}\text{"}\right) - 1 = -\infty - 1$

$= -\infty$

c.) $\lim\limits_{x \to 1^+} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = 1 - \left(\text{"}\frac{1}{0^+}\text{"}\right)$

$= 1 - \infty = -\infty$

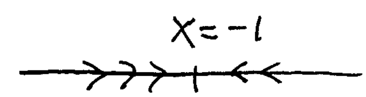
d.) $\lim\limits_{x \to 1^-} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = 1 - \left(\text{"}\frac{1}{0^+}\text{"}\right)$

$= 1 - \infty = -\infty$

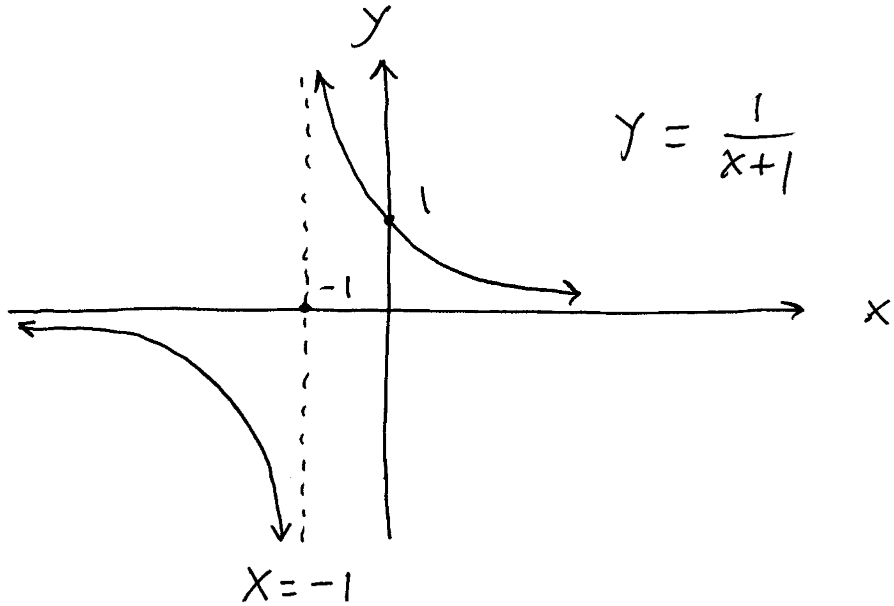
28.) $y = \frac{1}{x+1}$; $x = 0$: $\underline{Y = 1}$
$Y = 0$ (impossible) ;

$\lim\limits_{x \to \pm\infty} \frac{1}{x+1} = \text{"}\frac{1}{\pm\infty}\text{"} = 0$ so $\boxed{H.A.: Y = 0}$;

$\lim\limits_{x \to -1^+} \frac{1}{x+1} = \text{"}\frac{1}{0^+}\text{"} = +\infty$,



$\lim\limits_{x \to -1^-} \frac{1}{x+1} = \text{"}\frac{1}{0^-}\text{"} = -\infty$, so $\boxed{V.A.: X = -1}$;



$$Y = \frac{1}{x+1}$$

31.) $y = \dfrac{x+3}{x+2}$; $\quad x = 0 : \underline{Y = 3/2}$,

$Y = 0 : \dfrac{x+3}{x+2} = 0 \rightarrow x+3 = 0 \rightarrow \underline{x = -3}$;

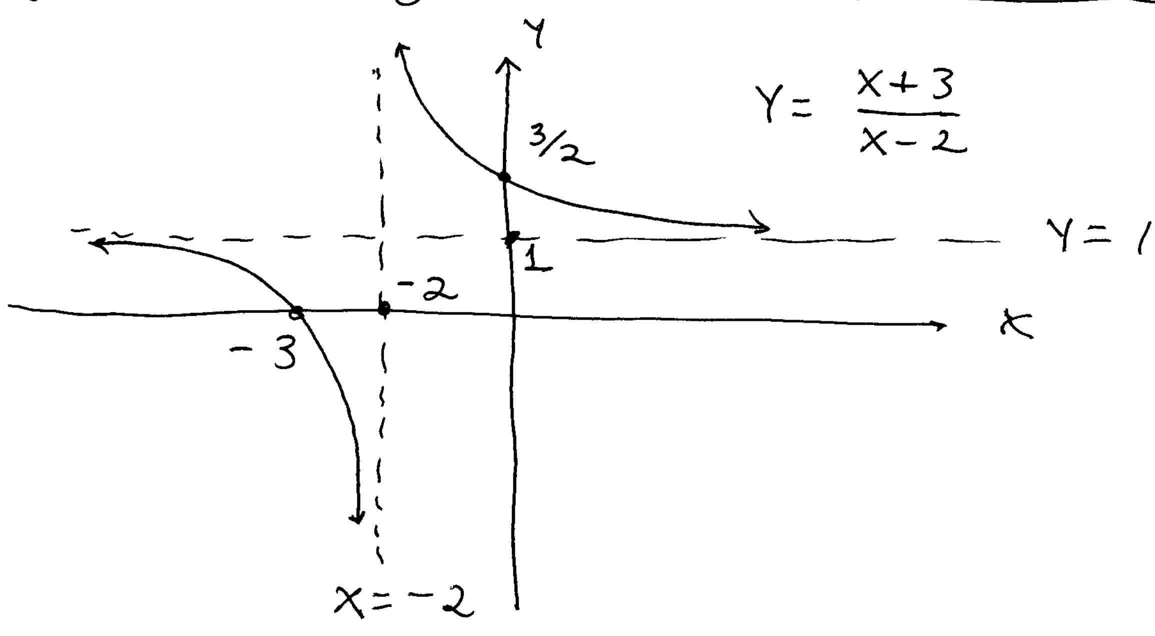
$$\lim_{x \to \pm\infty} \frac{x+3}{x+2} = \lim_{x \to \pm\infty} \frac{x+3}{x+2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \pm\infty} \frac{1 + 3/x}{1 + 2/x}$$

$= \dfrac{1+0}{1+0} = 1$ so $\boxed{H.A. : Y = 1}$;

$\lim\limits_{x \to -2^+} \dfrac{x+3}{x+2} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$,

$x = -2$

$\lim\limits_{x \to -2^-} \dfrac{x+3}{x+2} = \text{"}\dfrac{1}{0^-}\text{"} -\infty$, so $\boxed{V.A. : x = -2}$;



$$Y = \frac{x+3}{x-2}$$

34.) $Y = \dfrac{x^2+1}{x-1}$; $x=0$: $\underline{Y=-1}$,

$Y=0$: $\dfrac{x^2+1}{x-1} = 0 \rightarrow$

$x^2+1=0$ (impossible) ;

$$\lim_{x\to\infty} \frac{x^2+1}{x-1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{x+1/x}{1-1/x} = \frac{\infty+0}{1-0} = \infty ;$$

$$\lim_{x\to-\infty} \frac{x^2+1}{x-1}\cdot\frac{1/x}{1/x} = \lim_{x\to-\infty}\frac{x+1/x}{1-1/x} = \frac{-\infty+0}{1-0} = -\infty ;$$

$\boxed{\text{No H.A.}}$

$$\lim_{x\to1^+}\frac{x^2+1}{x-1} = \text{"}\frac{2}{0^+}\text{"} = +\infty ,$$

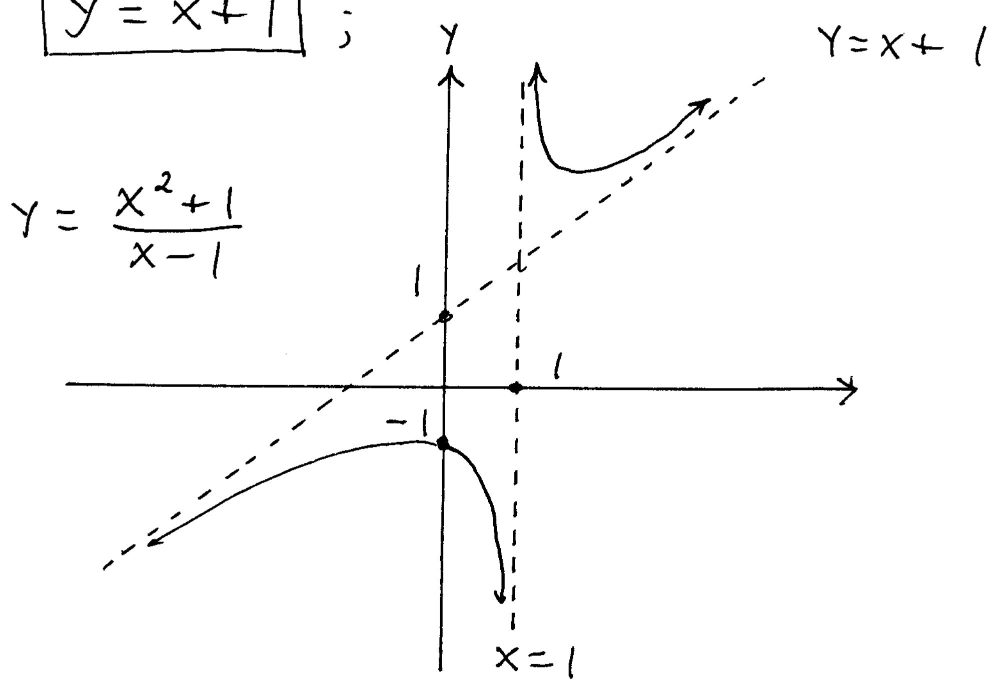
$$\lim_{x\to1^-}\frac{x^2+1}{x-1} = \text{"}\frac{2}{0^-}\text{"} = -\infty , \text{ so}$$

$\boxed{\text{V.A. : } x=1}$;

$$\begin{array}{r} x+1 \\ x-1 \overline{)\, x^2+1} \\ -(x^2-x) \\ \hline x+1 \\ -(x-1) \\ \hline 2 \end{array}$$

$$Y = \frac{x^2+1}{x-1} = x+1+\frac{2}{x-1}$$

so Tilted asymptote is $\boxed{y=x+1}$;



38.) $Y = \frac{x^3+1}{x^2}$; $x = 0$ (impossible) ;

$Y = 0$: $\frac{x^3+1}{x^2} = 0 \rightarrow x^3 + 1 = 0$

$\rightarrow x = -1$;

$\lim_{x \to \infty} \frac{x^3+1}{x^2} = \lim_{x \to \infty} \left(x + \frac{1}{x^2}\right) = \infty + 0 = \infty$;

$\lim_{x \to -\infty} \frac{x^3+1}{x^2} = \lim_{x \to -\infty} \left(x + \frac{1}{x^2}\right) = -\infty + 0 = -\infty$,

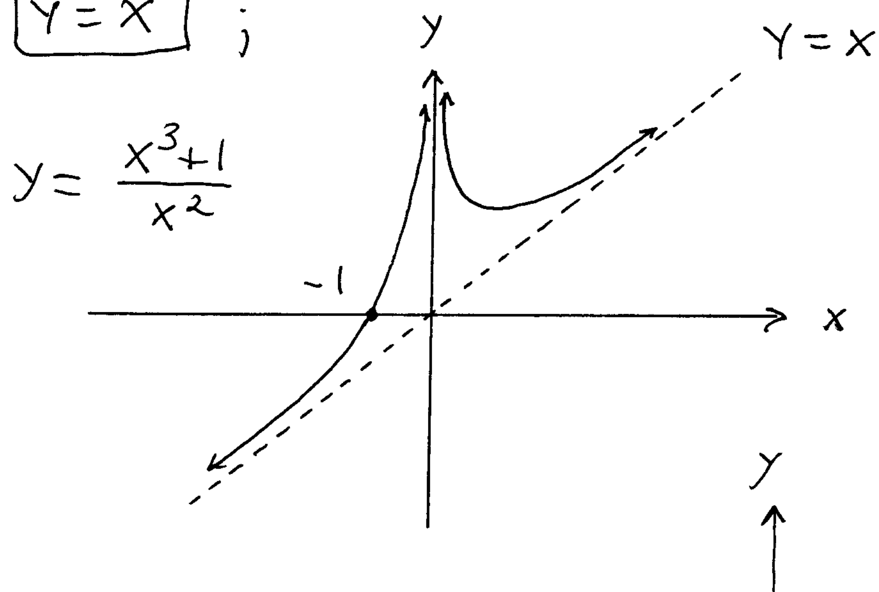
so $\boxed{\text{No H.A.}}$;

$\lim_{x \to 0^+} \frac{x^3+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$,

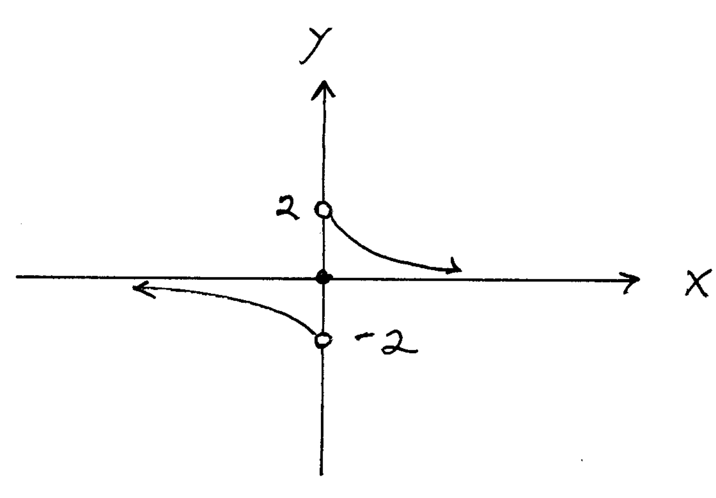
$\lim_{x \to 0^-} \frac{x^3+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$, so $\boxed{\text{V.A. : } x = 0}$;

$Y = \frac{x^3+1}{x^2} = x + \frac{1}{x^2}$, so <u>Tilted asymptote</u>

is $\boxed{Y = X}$;



40.)



41.)