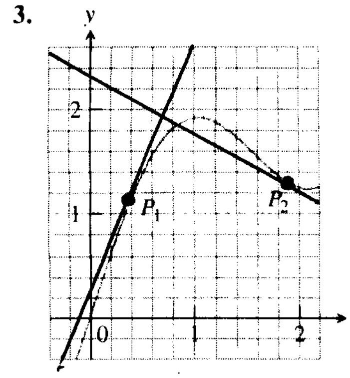
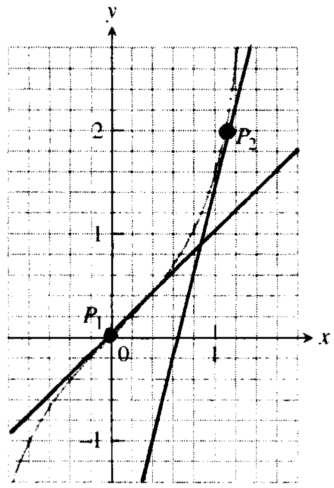
# Section 2.7

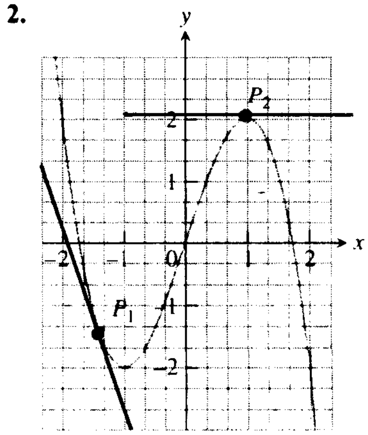
3.



1.



2.



1.) $P_1$ : slope $\approx \frac{9}{9} = 1$
$P_2$ : slope $\approx \frac{16}{4} = 4$

2.) $P_1$ : slope $\approx \frac{-9}{3} = -3$
$P_2$ : slope $\approx 0$

3.) $P_1$ : slope $\approx \frac{14}{6} = \frac{7}{3}$

$P_2$ : slope $\approx \frac{-5}{9}$

5.) $f(x) = 4 - x^2$ so

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(4-(x+h)^2) - (4-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} - \cancel{x^2} - 2hx - h^2 - \cancel{4} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x \text{ , i.e.,}$$

$f'(x) = -2x$ ; so slope of tangent line at $(-1, 3)$ is $m = f'(-1) = 2$ and line is given by

$Y - 3 = 2(x - (-1)) \rightarrow Y - 3 = 2x + 2 \rightarrow$

$\boxed{Y = 2x + 5}$.

8.) $f(x) = \frac{1}{x^2}$ so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h/1}$$

$$= \lim_{h \to 0} \frac{x^2 - (x+h)^2}{(x+h)^2 \cdot x^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2hx - h^2}{(x+h)^2 \cdot x^2 \cdot h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{(x+h)^2 \cdot x^2 \cdot \cancel{h}} = \frac{-2x}{x^2 \cdot x^2} \text{, i.e.,}$$

$f'(x) = \frac{-2}{x^3}$ ; so slope of tangent line at $(-1, 1)$ is $m = f'(-1) = \frac{-2}{(-1)^3} = 2$ and line is given by

$Y - 1 = 2(x - (-1)) \rightarrow Y - 1 = 2x + 2 \rightarrow$

$\boxed{Y = 2x + 3}$

12.) $f(x) = x - x^2$ so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left((x+h) - (x+h)^2\right) - (x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x^2} - 2hx - h^2 - \cancel{x} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(1 - 2x - h)}{\cancel{h}} = 1 - 2x \text{ , i.e.,}$$

$f'(x) = 1 - 2x$ ; so slope of tangent line at $(1,-1)$ is $m = f'(1) = 1 - 2 = -1$ and line is given by $Y - (-1) = -1(x - 1) \rightarrow$ $Y + 1 = -x + 1 \rightarrow \boxed{Y = -x}$ .

18.) $f(x) = \sqrt{x+1}$ so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h + \cancel{1} - \cancel{x} - \cancel{1}}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\ 1}{\cancel{h}\left(\sqrt{x+h+1} + \sqrt{x+1}\right)} = \frac{1}{\sqrt{x+1} + \sqrt{x+1}} ,$$

i.e., $f'(x) = \frac{1}{2\sqrt{x+1}}$ ; so slope of tangent line at $(8,3)$ is $f'(8) = \frac{1}{2\sqrt{9}} = 1/6$ and line is given by

$$Y - 3 = \frac{1}{6}(x - 8) \rightarrow Y - 3 = \frac{1}{6}x - \frac{4}{3} \rightarrow \boxed{Y = \frac{1}{6}x + \frac{5}{3}}$$

22.) $f(x) = \dfrac{x-1}{x+1}$ so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)-1}{(x+h)+1} - \frac{x-1}{x+1}}{\frac{h}{1}}$$

$$= \lim_{h \to 0} \frac{(x+h-1)(x+1) - (x+h+1)(x-1)}{(x+h+1)\ (x+1)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + hx - \cancel{x} + \cancel{x} + h - 1 - (x^2 + hX + \cancel{x} - \cancel{x} - h - 1)}{(x+h+1)\ (x+1)\ h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{hx} + \cancel{h} - \cancel{1} - \cancel{x^2} - \cancel{hx} + \cancel{h} + \cancel{1}}{(x+h+1)(x+1)\ h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{(x+h+1)(x+1)\ \cancel{h}} = \frac{2}{(x+1)^2}, \text{ i.e.,}$$

$f'(x) = \dfrac{2}{(x+1)^2}$ ; so slope of tangent line

at $x=0$ (and $Y=-1$) is $m = f'(0) = 2$

and line is given by

$Y-(-1) = 2(X-0) \rightarrow \boxed{Y = 2X-1}$

24.) $g(x) = x^3 - 3x$ so

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{((x+h)^3 - 3(x+h)) - (x^3 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{3x} - 3h - \cancel{x^3} + \cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3hx + h^2 - 3)}{\cancel{h}} = 3x^2 - 3, \text{ i.e.,}$$

$g'(x) = 3x^2 - 3$; horizontal tangent (slope $= 0$) means $g'(x) = 0$, i.e., $3x^2 - 3 = 0 \rightarrow 3(x-1)(x+1) = 0 \rightarrow$

$\underline{x = 1,\ y = -2}$ or $\underline{x = -1,\ y = 2}$.

31.) $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$;

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - 0}{h} = \lim_{h\to 0} \frac{h^2 \sin(1/h)}{h}$$

$$= \lim_{h\to 0} h \sin(1/h) \qquad \Bigg( -1 \leq \sin\left(\frac{1}{h}\right) \leq +1$$

$$\rightarrow \begin{cases} -h \leq h \sin(1/h) \leq h, & \text{if } h > 0 \\ -h \geq h \sin(1/h) \geq h, & \text{if } h < 0 \end{cases}; \text{ but}$$

$\lim_{h\to 0} -h = 0 = \lim_{h\to 0} h$, so by Squeeze Principle $\lim_{h\to 0} h \sin(1/h) = 0 \Bigg)$

$= 0$, i.e., $f'(0) = 0$.

32.) $g(x) = \begin{cases} x \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$;

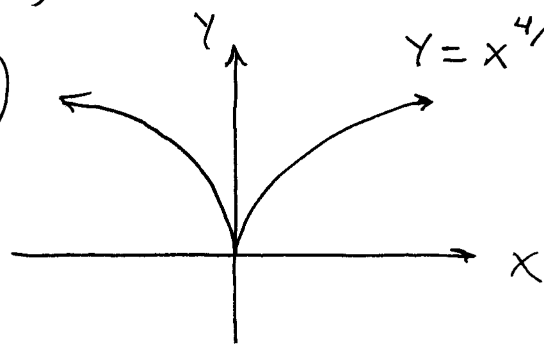
$$g'(0) = \lim_{h \to 0} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h \to 0} \frac{g(h) - 0}{h} = \lim_{h \to 0} \frac{\cancel{h} \sin(1/h)}{\cancel{h}}$$

$$= \lim_{h \to 0} \sin\left(\frac{1}{h}\right) \text{ DNE (oscillation between } -1 \text{ and } +1\text{)},$$

i.e., $g'(0)$ DNE .

36.) $Y = x^{4/5}$

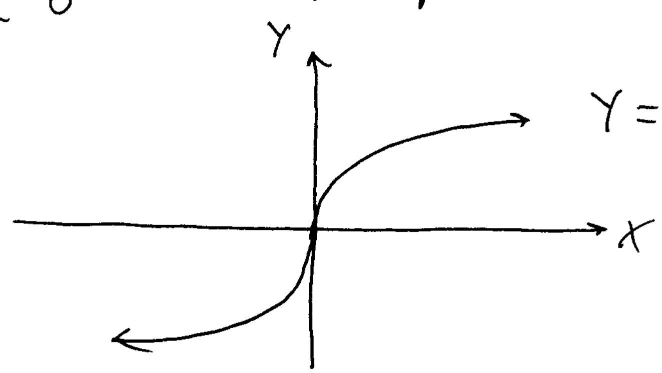



$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - 0}{h} = \lim_{h \to 0} \frac{h^{4/5}}{h} = \lim_{h \to 0} \frac{1}{h^{1/5}}$$

$$= \begin{cases} \text{“}\frac{1}{0^+}\text{”} = +\infty, & \text{if } h > 0 \\ \text{“}\frac{1}{0^-}\text{”} = -\infty, & \text{if } h < 0 \end{cases}, \text{ so } f'(0) \text{ DNE (corner)}$$

37.) $Y = x^{1/5}$




$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h^{1/5}}{h} = \lim_{h \to 0} \frac{1}{h^{4/5}} = \text{"}\frac{1}{0^+}\text{"} = +\infty,$$

so $f'(0)$ DNE (vertical tangent line)

# Section 3.1

6.) $r(s) = \sqrt{2s+1}$ so

$$r'(s) = \lim_{h \to 0} \frac{r(s+h) - r(s)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(s+h)+1} - \sqrt{2s+1}}{h} \cdot \frac{\sqrt{2s+2h+1} + \sqrt{2s+1}}{\sqrt{2s+2h+1} + \sqrt{2s+1}}$$

$$= \lim_{h \to 0} \frac{(\cancel{2s} + 2h + \cancel{1}) - (\cancel{2s} + \cancel{1})}{h \cdot (\sqrt{2s+2h+1} + \sqrt{2s+1})}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2s+2h+1} + \sqrt{2s+1})} = \frac{\cancel{2}}{\cancel{2}\sqrt{2s+1}}, \text{ i.e.,}$$

$$r'(s) = \frac{1}{\sqrt{2s+1}} \text{ ; then}$$

$$r'(0) = 1, \quad r'(1) = \frac{1}{\sqrt{3}}, \quad r'\left(\tfrac{1}{2}\right) = \frac{1}{\sqrt{2}} .$$

11.) $p = \dfrac{1}{\sqrt{q+1}}$ so

$$\frac{dp}{dq} = \lim_{h \to 0} \frac{p(q+h) - p(q)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{1}{\sqrt{q+h+1}} - \dfrac{1}{\sqrt{q+1}}}{h} .$$

$$= \lim_{h \to 0} \frac{\sqrt{q+1} - \sqrt{q+h+1}}{\sqrt{q+h+1} \cdot \sqrt{q+1}} \cdot \frac{1}{h} \cdot \frac{\sqrt{q+1} + \sqrt{q+h+1}}{\sqrt{q+1} + \sqrt{q+h+1}}$$

$$= \lim_{h \to 0} \frac{(\cancel{q} + \cancel{1}) - (\cancel{q} + h + \cancel{1})}{\sqrt{q+h+1} \cdot \sqrt{q+1} \cdot h\,(\sqrt{q+1} + \sqrt{q+h+1})}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\sqrt{q+h+1} \cdot \sqrt{q+1} \cdot \cancel{h} \cdot (\sqrt{q+1} + \sqrt{q+h+1})}$$

$$= \frac{-1}{\sqrt{q+1}\cdot\sqrt{q+1}\,\left(\sqrt{q+1}+\sqrt{q+1}\right)}$$

$$= \frac{-1}{(q+1)\cdot 2\sqrt{q+1}} = \frac{-1}{2(q+1)^{3/2}},$$ i.e., $\frac{dp}{dq} = \frac{-1}{2(q+1)^{3/2}}$

13.) $f(x) = x + \frac{9}{x}$ so

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left((x+h) + \frac{9}{x+h}\right) - \left(x + 9/x\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + h + \frac{9}{x+h} - \cancel{x} - \frac{9}{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\cancel{h}}{\cancel{h}} + \frac{\frac{9}{x+h} - \frac{9}{x}}{h/1}\right)$$

$$= \lim_{h\to 0} \left(1 + \frac{9x - 9(x+h)}{(x+h)x}\cdot\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(1 + \frac{\cancel{9x} - \cancel{9x} - 9h}{(x+h)x\cdot h}\right)$$

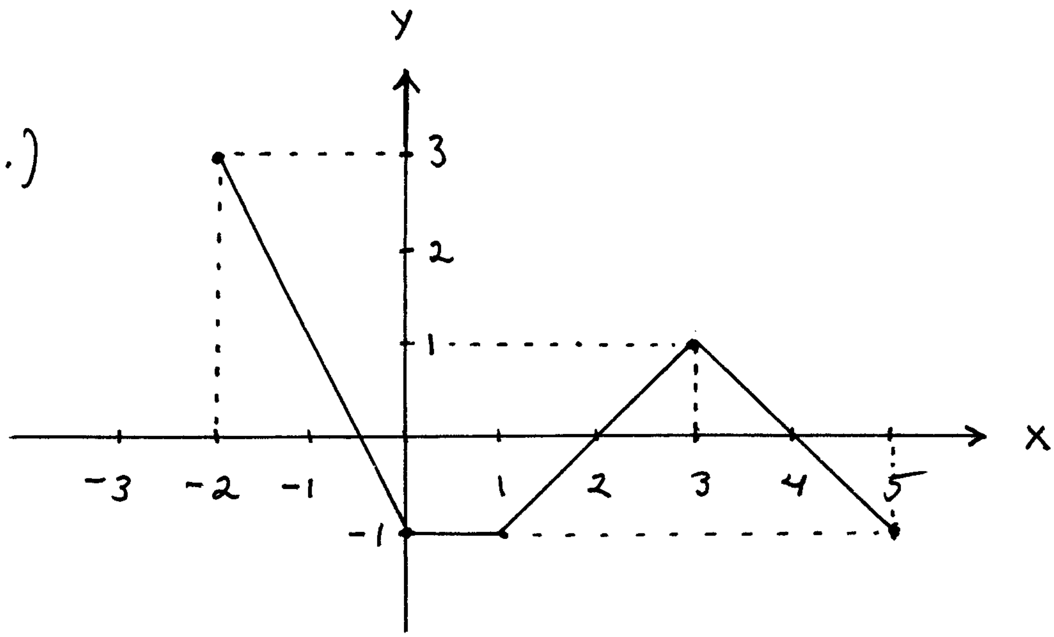
$$= \lim_{h\to 0} \left(1 - \frac{9\cancel{h}}{(x+h)x\cdot\cancel{h}}\right) = 1 - \frac{9}{x^2},$$ i.e.,

$$f'(x) = 1 - \frac{9}{x^2} \quad \text{so} \quad f'(-3) = 1 - \frac{9}{9} = 0$$

27.) b.) 28.) a.)

29.) d.) 30.) c.)

32.) a.)




40.) a.) diff. for $-2 \le x \le 3$

b.) cont., but not diff., for no x-values

c.) neither cont. nor diff. for no x-values

42.) a.) diff for $-2 \le x < -1$, $-1 < x < 0$, $0 < x < 2$, $2 < x < 3$

b.) cont., but not diff., for $x = -1$

c.) neither cont. nor diff. for $x = 0$, $x = 2$

43.) a.) diff. for $-1 \le x < 0$, $0 < x \le 2$

b.) cont., but not diff., for $x = 0$

c.) neither cont. nor diff. for no x-values