

Section 3.3

$$1.) \quad s = t^2 - 3t + 2 = (t-2)(t-1),$$

$$\text{vel. } s' = 2t - 3 = 2\left(t - \frac{3}{2}\right),$$

$$\text{acc. } s'' = 2 \quad ; \quad 0 \leq t \leq 2$$

$$a.) \quad \text{displ.} = s(2) - s(0) = 0 - 2 = -2 \text{ m.};$$

$$\text{ave. vel.} = \frac{s(2) - s(0)}{2 - 0} = \frac{-2}{2} = -1 \text{ m./s.}$$

$$b.) \quad \text{speed} = |s'(0)| = |1 - 3| = 2 \text{ m./s.},$$

$$\text{speed} = |s'(2)| = |1| = 1 \text{ m./s.},$$

$$\text{acc.} = s''(0) = 2 \text{ m./s.}^2,$$

$$\text{acc.} = s''(2) = 2 \text{ m./s.}^2,$$

$$c.) \quad s'(t) = 2\left(t - \frac{3}{2}\right) = 0 \rightarrow$$

vel. $s' = 0$ m./s. when $t = \frac{3}{2}$ sec.
and object changes direction
(moving left to moving right)

$$4.) \quad s = \frac{1}{4}t^4 - t^3 + t^2$$

$$= \frac{1}{4}t^2(t^2 - 4t + 4) = \frac{1}{4}t^2(t-2)^2,$$

$$\text{vel. } s' = t^3 - 3t^2 + 2t = t(t-1)(t-2),$$

$$\text{acc. } s'' = 3t^2 - 6t + 2 \quad ; \quad 0 \leq t \leq 3$$

$$a.) \quad \text{displ.} = s(3) - s(0) = \frac{9}{4} - 0 = \frac{9}{4} \text{ m.},$$

$$\text{ave. vel.} = \frac{s(3) - s(0)}{3 - 0} = \frac{\frac{9}{4}}{\frac{3}{1}} = \frac{9}{4} \cdot \frac{1}{3} = \frac{3}{4} \text{ m./s.}$$

$$b.) \quad \text{speed} = |s'(0)| = |0| = 0 \text{ m./s.}$$

$$\begin{aligned} \text{speed} &= |s'(3)| = |6| = 6 \text{ m./s.} \\ \text{acc} &= \mp s''(0) = 2 \text{ m./s.}^2 \\ \text{acc} &= s''(3) = 11 \text{ m./s.}^2 \end{aligned}$$

c.) vel. $s' = t(t-1)(t-2) = 0 \rightarrow$
 $t=0, t=1, \text{ or } t=2 \text{ sec.}$; when
 $\boxed{t=1}$ sec. $s'(1)=0$ and object
 changes direction (moving
 right to moving left); when
 $\boxed{t=2}$ sec. $s'(2)=0$ and object
 changes direction (moving
 left to moving right)

$$\begin{aligned} 7.) \quad s &= t^3 - 6t^2 + 9t = t(t-3)^2, \\ \text{vel. } s' &= 3t^2 - 12t + 9 \\ &= 3(t^2 - 4t + 3) = 3(t-1)(t-3), \\ \text{acc. } s'' &= 6t - 12 = 6(t-2) \end{aligned}$$

a.) vel. $s' = 0 \rightarrow 3(t-1)(t-3) = 0 \rightarrow$
 $\boxed{t=1}$ sec., $\boxed{t=3}$ sec. ;

$$\text{acc. } s''(1) = -6 \text{ m./s.}^2,$$

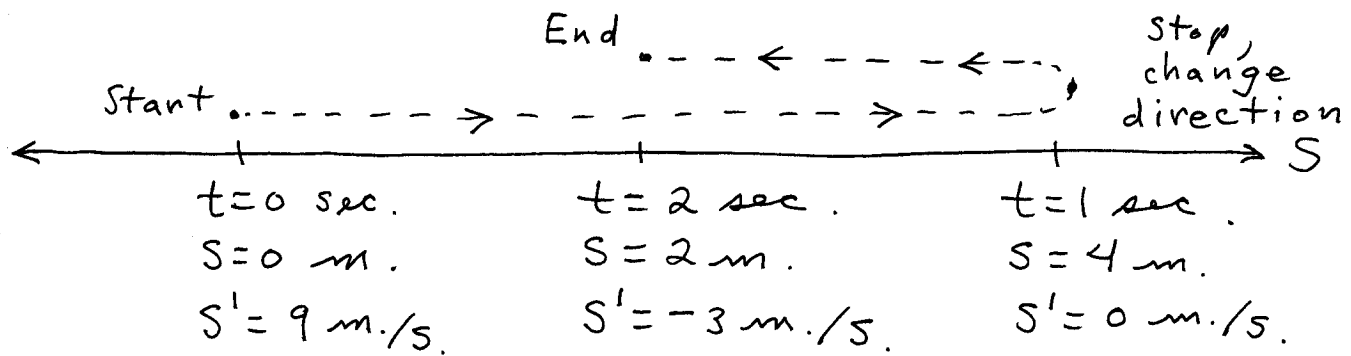
$$\text{acc. } s''(3) = 6 \text{ m./s.}^2$$

b.) acc. $s'' = 0 \rightarrow 6(t-2) = 0 \rightarrow \boxed{t=2}$ sec.

$$\rightarrow \text{vel. } s'(2) = -3 \text{ m./s.}$$

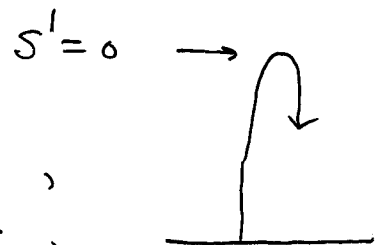
c.) $s(0) = 0 \text{ m. and } s'(0) = 9 \text{ m./s.}$

so object starts at the origin and begins moving to the right;



the total distance traveled is

$$4 + 2 = 6 \text{ m.}$$



10.) a.) $S = 24t - 0.8t^2 \text{ m.}$

vel. $S' = 24 - 1.6t \text{ m./s.}$

acc. $S'' = -1.6 \text{ m./s}^2$

b.) highest point: $S' = 0 \rightarrow$

$$24 - 1.6t = 0 \rightarrow t = 15 \text{ sec.}$$

c.) $S(15) = 24(15) - 0.8(15)^2 = 180 \text{ m.}$

d.) If $S = 90 \text{ m.} \rightarrow 90 = 24t - \frac{4}{5}t^2 \rightarrow$

$$\frac{4}{5}t^2 - 24t + 90 = 0 \rightarrow 4t^2 - 120t + 450 = 0$$

$$\rightarrow 2(2t^2 - 60t + 225) = 0 \rightarrow$$

$$t = \frac{60 \pm \sqrt{3600 - 4(2)(225)}}{4} \approx \frac{60 \pm 42.4}{4} \rightarrow$$

$t \approx 4.4 \text{ sec.}$ or $t \approx 25.6 \text{ sec. (No)}$

e.) $S = 0 \rightarrow 24t - 0.8t^2 = 0 \rightarrow$

$$t(24 - 0.8t) = 0 \rightarrow t = 0 \text{ sec. or}$$

$$24 - 0.8t = 0 \rightarrow \text{rock is aloft for}$$

$$\boxed{t = 30} \text{ sec.}$$

$$12.) \quad s = 832t - 2.6t^2 \quad (\text{moon})$$

$$h = 832t - 16t^2 \quad (\text{earth})$$

$$a.) \quad s = 0 \rightarrow 832t - 2.6t^2 = 0 \rightarrow$$

$$t(832 - 2.6t) = 0 \rightarrow t = 0 \text{ or}$$

$$832 - 2.6t = 0 \rightarrow \text{bullet is aloft for}$$

$$\boxed{t = 320} \text{ sec. ; } s' = 0 \rightarrow 832 - 5.2t = 0$$

$$\rightarrow t = 160 \text{ sec.} \rightarrow \text{height of bullet}$$

$$\text{is } s(160) = \underline{66,560 \text{ ft.}}$$

$$b.) \quad h = 0 \rightarrow 832t - 16t^2 = 0 \rightarrow$$

$$t(832 - 16t) = 0 \rightarrow t = 0 \text{ or}$$

$$832 - 16t = 0 \rightarrow \text{bullet is aloft for}$$

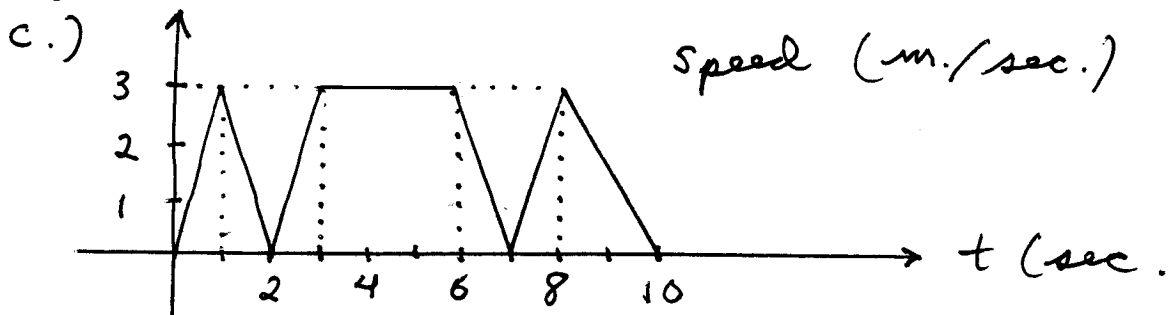
$$\boxed{t = 52} \text{ sec. ; } h' = 0 \rightarrow 832 - 32t = 0$$

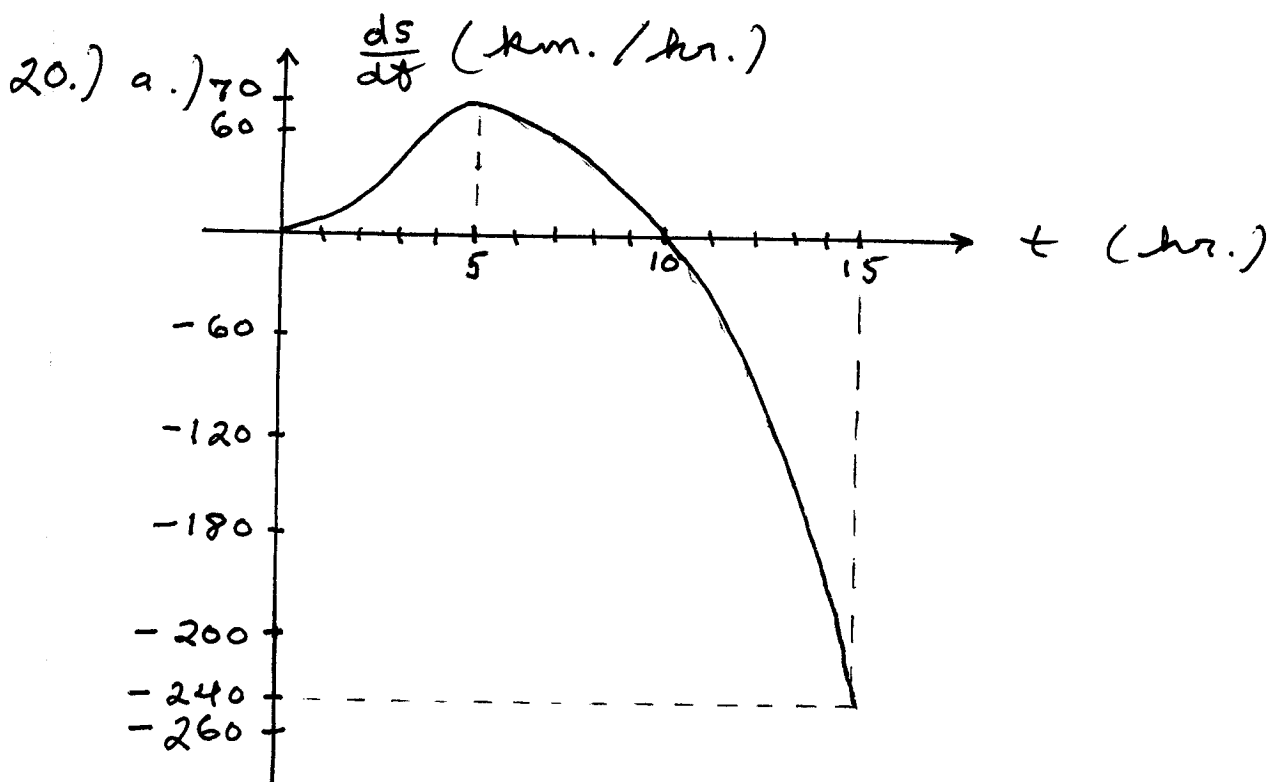
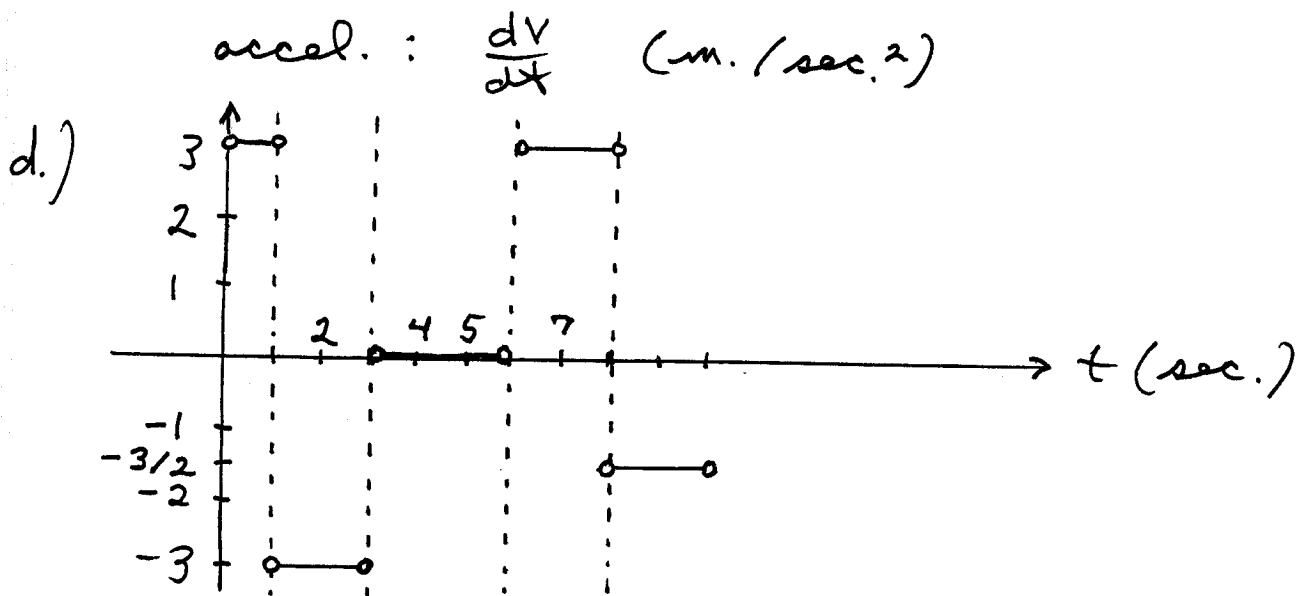
$$\rightarrow t = 26 \text{ sec.} \rightarrow \text{height of bullet}$$

$$\text{is } h(26) = \underline{10,816 \text{ ft.}}$$

$$15.) \quad a.) \quad t = 2 \text{ sec.}, \quad t = 7 \text{ sec.}$$

$$b.) \quad 3 \leq t \leq 6 \text{ sec.}$$





26.) $Q(t) = 200(30-t)^2 = 200(t^2 - 60t + 900)$

$\frac{D}{\rightarrow} \frac{dQ}{dt} = 200(2t - 60)$ and $t = 10$ min. \rightarrow

$\frac{dQ}{dt} = 200(2(10) - 60) = -8000$ gal./min. ;

Average rate = $\frac{Q(10) - Q(0)}{10 - 0} = \frac{80,000 - 180,000}{10}$

= -10,000 gal./min.

29.) Distance $D = \frac{10}{9} t^2$ m. \rightarrow

Velocity $v = \frac{dD}{dt} = \frac{20}{9} t$ m./sec. j

if $v = \frac{200 \text{ km.}}{1 \text{ hr.}} \cdot \frac{1 \text{ hr.}}{3600 \text{ sec.}} \cdot \frac{1000 \text{ m.}}{1 \text{ km}}$
 $= \frac{500}{9}$ m./sec., then

$\frac{500}{9} = \frac{20}{9} t \rightarrow \boxed{t = 25}$ sec. and

$D = \frac{10}{9} (25)^2 \approx 694.4$ m.