Math 21D
Vogler
Discussion Sheet 10

1.) Find the area of the following surfaces S, which are directly above the rectangular region R with vertices $(0,0)$, $(2,0)$, $(2,4)$, and $(0,4)$ in the xy-plane.

a.) plane $z = 5$
b.) plane $z = 2y$
c.) plane $x + 2y + 3z = 12$

2.) Find the area of the following surfaces S, which are directly above the disc $x^2 + y^2 \leq 9$ in the xy-plane.

a.) top half of sphere $x^2 + y^2 + z^2 = 64$
b.) paraboloid $z = x^2 + y^2 + 1$
c.) cone $z = \sqrt{x^2 + y^2}$

3.) Let surface S be the top half of the sphere $x^2 + y^2 + z^2 = 4$. Define the following function g on S : $g(P) = g(x, y, z)$ is the square of the distance from P to the xy-plane. Compute the surface integral of g over S.

4.) Let surface S be that portion of the paraboloid $z = x^2 + y^2 + 4$ directly above the disc $x^2 + y^2 \leq 1$ in the xy-plane. Let function $g(x, y, z) = \sqrt{x^2 + y^2}$. Compute the surface integral of g over S.

5.) Let surface S be that portion of the paraboloid $z = 4 - x^2 - y^2$ cut by the plane $z = 0$. Find the Flux of the vector field $\vec{F}(x, y, z) = (y)\vec{i} + (x)\vec{j} + (z)\vec{k}$ outward through the surface S.

6.) Find the Flux of the vector field $\vec{F}(x, y, z) = (2x)\vec{i} + (-3y)\vec{j} + (z)\vec{k}$ in the direction away from the origin and across the region S in the plane $x + 2y + 3z = 12$, which is directly above the triangle with vertices $(0,0)$, $(0,2)$, and $(2,6)$ in the xy-plane.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

7.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet ?

1.) Compute the divergence of $\vec{F}$ and the curl of $\vec{F}$ for each of the following vector fields.

a.) $\vec{F}(x,y,z) = (x^4)\vec{i} + (-x^3z^2)\vec{j} + (4xy^2z)\vec{k}$
b.) $\vec{F}(x,y,z) = (xy\sin z)\vec{i} + (\cos(xz))\vec{j} + (y\cos z)\vec{k}$

2.) Verify Stoke's Theorem for $\vec{F}(x,y,z) = (y^2)\vec{i} + (x)\vec{j} + (z^2)\vec{k}$, where surface S is that portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

3.) Use Stoke's Theorem to evaluate $\int\int_S \vec{\nabla} \times \vec{F} \cdot \vec{n}\ dS$, where $\vec{F}(x,y,z) = (yz)\vec{i} + (xz)\vec{j} + (xy)\vec{k}$ and surface S is that portion of the paraboloid $z = 9 - x^2 - y^2$ above the plane $z = 5$.

4.) Use Stoke's Theorem to evaluate $\oint_C \vec{F} \cdot \vec{T}\ ds$, where $\vec{F}(x,y,z) = (e^{-x})\vec{i} + (e^x)\vec{j} + (e^z)\vec{k}$ and surface S is that portion of the plane $2x + y + 2z = 2$ in the first octant.

5.) Verify the Divergence Theorem for $\vec{F}(x,y,z) = (xy)\vec{i} + (yz)\vec{j} + (xz)\vec{k}$, where the solid D is the cylinder $x^2 + y^2 = 1$ for $0 \le z \le 1$.

6.) Use the Divergence Theorem to evaluate $\int\int_S \vec{F} \cdot \vec{n}\ dS$, where $\vec{F}(x,y,z) = (e^x \sin y)\vec{i} + (e^x \cos y)\vec{j} + (yz^2)\vec{k}$ and surface S is the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$.

7.) Use the Divergence Theorem to evaluate $\int\int\int_D div\ \vec{F}\ dV$, where $\vec{F}(x,y,z) = (xe^y)\vec{i} + (xz)\vec{j} + (x\sin z)\vec{k}$ and the solid D is the cube with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

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"An individual has not started living until he can rise above the narrow confines of his individualistic concerns to the broader concerns of all humanity." – Martin Luther King, Jr.

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