

Math 21D  
Vogler  
Discussion Sheet 3

- 1.) Consider a flat plate lying in the region bounded by the graphs of  $y = e^x$ ,  $x = 0$ , and  $y = 2$ . Assume that density at point  $(x, y)$  is given by  $\delta(x, y) = x^2y^3 + 1$ .
- Set up but do not evaluate a double integral which represents the area of the plate.
  - Set up but do not evaluate a double integral which represents the mass of the plate.
  - Set up but do not evaluate double integrals which represent the centroid of the plate.
  - Set up but do not evaluate double integrals which represent the center of mass of the plate.
  - Set up but do not evaluate double integrals which represent the moment of inertia of the plate about
    - the origin.
    - the x-axis.
    - the line  $x = 4$ .
- 2.) Consider a flat plate lying in the region bounded by the graphs of  $x = y^2$  and  $x = 2 - y$ . Assume that density at point  $(x, y)$  is given by  $\delta(x, y) = \ln(x^2y^2 + 4)$ .
- Set up but do not evaluate a double integral which represents the area of the plate.
  - Set up but do not evaluate a double integral which represents the mass of the plate.
  - Set up but do not evaluate double integrals which represent the centroid of the plate.
  - Set up but do not evaluate double integrals which represent the center of mass of the plate.
  - Set up but do not evaluate double integrals which represent the moment of inertia of the plate about
    - the origin.
    - the y-axis.
    - the line  $y = -3$ .
- 3.) Consider region  $R$  bounded by the graphs of  $x = y^3$ ,  $x = 2$ , and  $y = 0$ . Find the
- average height of region  $R$ .
  - average width of region  $R$ .
  - average distance from points  $(x, y)$  in  $R$  to the point  $(0, 4)$ .
- 4.) Let  $R$  be the region in the first quadrant on or inside the circle  $x^2 + y^2 = 9$ .
- Describe  $R$  using vertical cross-sections.
  - Describe  $R$  using horizontal cross-sections.
  - Describe  $R$  using polar coordinates in the format

- i.)  $a \leq \theta \leq b, f(\theta) \leq r \leq g(\theta)$
- ii.)  $a \leq r \leq b, f(r) \leq \theta \leq g(r)$

5.) Let  $R$  be the region bounded by the graphs of  $y = x, x = 0$ , and  $y = 3$ .

- a.) Describe  $R$  using vertical cross-sections.
- b.) Describe  $R$  using horizontal cross-sections.
- c.) Describe  $R$  using polar coordinates in the format
  - i.)  $a \leq \theta \leq b, f(\theta) \leq r \leq g(\theta)$
  - ii.)  $a \leq r \leq b, f(r) \leq \theta \leq g(r)$

6.) Let  $R$  be the region on or inside the circle  $x^2 + (y - 2)^2 = 4$ .

- a.) Describe  $R$  using vertical cross-sections.
- b.) Describe  $R$  using horizontal cross-sections.
- c.) Describe  $R$  using polar coordinates in the format
  - i.)  $a \leq \theta \leq b, f(\theta) \leq r \leq g(\theta)$
  - ii.)  $a \leq r \leq b, f(r) \leq \theta \leq g(r)$

7.) Evaluate the following double integrals.

$$\begin{array}{ll} \text{a.) } \int_0^{\pi/2} \int_0^{\sin \theta} r \cos \theta \, dr \, d\theta & \text{b.) } \int_0^{\pi} \int_0^{1+\cos \theta} r \, dr \, d\theta \\ \text{c.) } \int_{-\pi/2}^{\pi/2} \int_0^{\sin \theta} r^2 \, dr \, d\theta & \text{d.) } \int_0^{\pi} \int_0^{1-\sin \theta} r^2 \cos \theta \, dr \, d\theta \end{array}$$

8.) For each of the following problems, sketch the two-dimensional region described by the iterated integral, convert to polar coordinates, and evaluate the double integral.

$$\begin{array}{ll} \text{a.) } \int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) \, dy \, dx & \text{b.) } \int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)} \, dx \, dy \\ \text{c.) } \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy \, dx & \text{d.) } \int_0^4 \int_3^{\sqrt{25-x^2}} dy \, dx \end{array}$$

9.) Use a double integral to find the area of the region in the first quadrant between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$ .

10.) Find the average distance from points  $(x, y)$  on or inside a circle of radius  $r$  to the center of the circle.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

11.) The minute and hour hand of a watch line up perfectly at 12 o'clock. In how many minutes and seconds will the hands line up perfectly again ?