

Math 21D
Vogler
Discussion Sheet 4

- 1.) Let R be the solid region bounded by the hemi-sphere $z = \sqrt{4 - x^2 - y^2}$ and the plane $z = 0$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid
- using rectangular coordinates.
 - using cylindrical coordinates.
- 2.) Let R be the solid region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the hemi-sphere $z = \sqrt{18 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid
- using rectangular coordinates.
 - using cylindrical coordinates.
- 3.) Let R be the solid region inside the cylinder $x^2 + y^2 = 4$ and bounded by the plane $z = 0$ and the hemi-sphere $z = \sqrt{9 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid
- using rectangular coordinates.
 - using cylindrical coordinates.
- 4.) Let R be the solid region enclosed by the paraboloid $z = 6 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$. If the temperature at point $P = (x, y, z)$ is given by $T = \ln(x^2 + y^2 + z^2 + 1)$, then SET UP BUT DO NOT EVALUATE triple integrals representing the average temperature of the solid
- using rectangular coordinates.
 - using cylindrical coordinates.
- 5.) Consider the UFO bounded by the surfaces $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. The density of the UFO at point $P = (x, y, z)$ is given by the square of the distance from P to the z -axis. SET UP BUT DO NOT EVALUATE triple integrals in rectangular and cylindrical coordinate systems which represent the UFO's
- average density.
 - total mass.
 - total volume.
 - x -coordinate of the centroid.
 - y -coordinate of the center of mass.
 - moment of inertia about
 - the origin.
 - the z -axis.
 - the line parallel to the x -axis and passing through the point $(3, -4, 5)$.

6.) Convert the following cylindrical integral to rectangular coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_0^{2\pi} \int_0^2 \int_0^{5-r \cos \theta - r \sin \theta} r^4 \sin^2 \theta \cos \theta \, dz \, dr \, d\theta$$

7.) Convert the following rectangular integral to cylindrical coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_{-2}^2 \int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} (x^2 + y^2)^3 \cos z \, dz \, dy \, dx$$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

8.) The camp cook wants to measure exactly four ounces of vinegar out of a jug, but has only a five-ounce container and a three-ounce container. How can the cook accomplish the task ?