Math 21D
Vogler
Discussion Sheet 5

1.) Let R be the solid region bounded by the surfaces $z = \sqrt{4 - x^2 - y^2}$ and $z = 0$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

2.) Let R be the solid region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{18 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

3.) Let R be the solid region inside the surface $x^2 + y^2 = 4$ and bounded by the surfaces $z = 0$ and $z = \sqrt{9 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

4.) Consider the chocolate chip cookie bounded by the surfaces $z = 9 - x^2 - y^2$ and $z = 9 - 3y$. The density of the cookie at point $P = (x, y, z)$ is given by one plus the distance from P to the point $(0, 0, 9)$. SET UP BUT DO NOT EVALUATE triple integrals which represent the cookie's total mass (yummy) using

a.) rectangular coordinates.
b.) cylindrical coordinates.
c.) spherical coordinates.

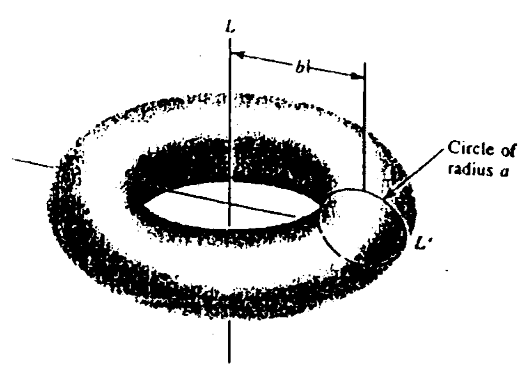
5.) Convert the following cylindrical integral to spherical coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_0^{2\pi} \int_2^{\sqrt{5}} \int_0^{\sqrt{5-r^2}} r^2 z \cos\theta \, dz \, dr \, d\theta$$

6.) Sketch the solid whose volume is given by the following spherical integral.

$$\int_0^{\pi} \int_0^{\pi/2} \int_2^{3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

7.) SET UP BUT DO NOT EVALUATE a triple integral which represents the volume of the given doughnut (torus).



8.) Let $F(u, v) = (u - v, 2u + v) = (x, y)$.

a.) Determine $F(0,0)$, $F(2,-3)$, and $F(4,1)$

b.) Let R be the line $v = u$ in the uv-plane. Find the image S of R under F. Sketch both R and S.

c.) Let R be an arbitrary line in the uv-plane, i.e., let $v = mu + b$. Prove that the image S of R under F is also a line.

d.) Let R be the rectangle with vertices $(0,0)$, $(2,0)$, $(0,3)$, and $(2,3)$ and all points in its interior. Find the image S of R under F. Sketch both R and S.

e.) Find the magnification $|J(P)|$ of F.

f.) Determine the area of region S in part d.).

9.) Let $F(u, v) = (u^2 + v^2, uv) = (x, y)$.

a.) Find $|J(-1,2)|$.

a.) Find $|J(4,3)|$.

a.) Find $|J(-3,-2)|$.

10.) Let $F(r, \theta) = (r\cos\theta, r\sin\theta) = (x, y)$. Find $|J(P)|$.

11.) Let $F(\rho, \phi, \theta) = (\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi) = (x, y, z)$. Find $|J(P)|$.

12.) Let $F(u, v) = (2v, 3u) = (x, y)$. Let R be the region in the uv-plane given by $u^2 + v^2 \leq 1$.

a.) Find the image S of R under F. Sketch both R and S.

b.) Find the area of S.

13.) Let $F(u, v) = (u + v, 2u - v) = (x, y)$. Let R be the region in the uv-plane given by $u^2 + v^2 \leq 1$.

a.) Let S be the image of R under F. S is a tilted non-standard ellipse and its interior, which you can sketch by plotting random points on its perimeter ! Find the area of S.

b.) Find a line in the xy-plane which passes through the origin and which splits the area of S in half.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

14.) A perfect square triangular (PST) number is a whole number with the the following properties. It is a perfect square and it is also equal to one-half the product of consecutive whole numbers. Find the first four PST numbers.