

Math 21D
Vogler
Discussion Sheet 7

- 1.) Show that the curve plotted by projectile motion,
 $\vec{r}(t) = (|\vec{v}_0| \cos \alpha \cdot t) \vec{i} + (|\vec{v}_0| \sin \alpha \cdot t - (1/2)gt^2) \vec{j}$, is a parabola in the xy -plane.
- 2.) Show that the maximum downrange distance for projectile motion with a given initial speed $|\vec{v}_0|$ occurs when $\alpha = 45^\circ$ and is $x = \frac{|\vec{v}_0|^2}{g}$ (Hint: See formula III on Projectile Motion Handout.).
- 3.) Consider path C plotted by vector function $\vec{r}(t) = t \vec{i} + \sqrt{t} \vec{j}$ for $0 \leq t \leq 9$.
 - a.) Sketch C .
 - b.) Find $\vec{v}(t)$, $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{a}(t)$.
 - c.) Plot $\vec{r}(1)$, $\vec{v}(1)$, $\vec{T}(1)$, $\vec{N}(1)$, and $\vec{a}(1)$. Also compute the speed and acceleration of motion when $t = 1$.
- 4.) Let $\vec{r}(t) = (12 \sin t) \vec{i} + (-12 \cos t) \vec{j} + (5t) \vec{k}$ be a position vector function.
 - a.) Determine the position vector when
 - i.) $t = 0$ ii.) $t = 7\pi/6$
 - b.) Write t as a function of arc length s . Write $\vec{r}(t)$ as a function of arc length s , i.e., write $\vec{r}(t) = \vec{r}(t(s))$.
 - c.) Determine the position vector when
 - i.) $s = 0$ ii.) $s = 39\pi$
- 5.) Determine the length of path C determined $\vec{r}(t) = (\cos^3 t) \vec{i} + (\sin^3 t) \vec{j}$ for $0 \leq t \leq 2\pi$.
- 6.) Evaluate the following line integrals.
 - a.) $\int_C 2x \, ds$, $\vec{r}(t) = (1/2)t^2 \vec{i} + (1/4)t^4 \vec{j}$ for $0 \leq t \leq 2$
 - b.) $\int_C \sqrt{x^2 + z^2} \, ds$, $\vec{r}(t) = (2 \cos t) \vec{j} + (2 \sin t) \vec{k}$ for $\pi \leq t \leq 2\pi$
 - c.) $\int_C 3xyz \, ds$, $\vec{r}(t) = t \vec{i} + 2t \vec{j} - t \vec{k}$ for $0 \leq t \leq 4$

7.) A spring lies on the path determined by $\vec{r}(t) = (\sin t) \vec{i} + (\cos t) \vec{j} + (2/3)t^{3/2} \vec{k}$ for $0 \leq t \leq 4\pi$. Sketch the wire and find its length.

8.) Find the area of the vertical wall sitting on the xy -plane on the line $y = 2x$ from $x = 0$ to $x = 4$, if the height of the wall at the point (x, y) is xy^2 .

9.) A wire lies on the path determined by the helix $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + t \vec{k}$ for $0 \leq t \leq 2\pi$. Its density at point (x, y, z) is given by $\delta(x, y, z) = xy + z + 3$. Compute the

- a.) length of the wire.
- b.) mass of the wire.
- c.) x -coordinate for its center of mass.
- d.) z -coordinate for its centroid.
- e.) moment of the wire relative to the plane $y = 1$.
- f.) moment of inertia of the wire about
 - i.) the origin
 - ii.) the z -axis

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) Divide the following figure into 4 parts each of the same size and shape.

