

Math 21C

Vogler

Problems Using (*) and (*) (*) from the Integral Test Handout

1.) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Use (*) to put a lower and an upper bound on the partial sum

- a.) s_{10} , the sum of the first 10 terms of this series.
- b.) s_{1000} , the sum of the first 1000 terms of this series.
- c.) $s_{1,000,000}$, the sum of the first 1,000,000 terms of this series.

2.) The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

- a.) Compute the partial sum $s_{10} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{10^2} = \sum_{i=1}^{10} \frac{1}{i^2}$.
- b.) Use (*) (*) to put a lower and an upper bound on the error (remainder)

$$R_{10} = \frac{1}{11^2} + \frac{1}{12^2} + \frac{1}{13^2} + \cdots \text{ for the partial sum } s_{10}.$$

- c.) Use (*) (*) to put a lower and an upper bound on the error (remainder)

$$R_{100} = \frac{1}{101^2} + \frac{1}{102^2} + \frac{1}{103^2} + \cdots \text{ for the partial sum } s_{100}.$$

- d.) What should n be so that the partial sum $s_n = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} = \sum_{i=1}^n \frac{1}{i^2}$

estimates the exact value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ with an error R_n of at most 0.0001 ?

3.) The series $\sum_{n=1}^{\infty} (2/3)^{n-1}$ converges.

- a.) Compute the partial sum $s_{10} = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^9 = \sum_{i=1}^{10} (2/3)^{i-1}$.
- b.) Use (*) (*) to put a lower and an upper bound on the error (remainder)

$$R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots .$$

- c.) Compute the exact value of $R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots .$
- d.) What should n be so that the partial sum

$$s_n = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^{n-1} = \sum_{i=1}^n (2/3)^{i-1} \text{ estimates the exact value of } \sum_{n=1}^{\infty} (2/3)^{n-1}$$

with an error R_n of at most 0.0001 ?

- e.) What is the exact value of the series $\sum_{n=1}^{\infty} (2/3)^{n-1}$?