

## Section 12.2

1.) a.)  $3\vec{u} = 3(3, -2) = (9, -6)$

b.)  $|3\vec{u}| = \sqrt{9^2 + (-6)^2} = \sqrt{117} = 3\sqrt{13}$

4.) a.)  $\vec{u} - \vec{v} = (3, -2) - (-2, 5) = (5, -7)$

b.)  $|\vec{u} - \vec{v}| = \sqrt{5^2 + (-7)^2} = \sqrt{74}$

6.) a.)  $-2\vec{u} + 5\vec{v} = -2(3, -2) + 5(-2, 5)$   
 $= (-6, 4) + (-10, 25) = (-16, 29)$

b.)  $|-2\vec{u} + 5\vec{v}| = \sqrt{(-16)^2 + (29)^2} = \sqrt{1097}$

7.) a.)  $\frac{3}{5}\vec{u} + \frac{4}{5}\vec{v} = \frac{3}{5}(3, -2) + \frac{4}{5}(-2, 5)$   
 $= \left(\frac{9}{5}, -\frac{6}{5}\right) + \left(-\frac{8}{5}, 4\right) = \left(\frac{1}{5}, \frac{14}{5}\right)$

b.)  $\left|\frac{3}{5}\vec{u} + \frac{4}{5}\vec{v}\right| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{14}{5}\right)^2} = \sqrt{\frac{197}{25}} = \frac{\sqrt{197}}{5}$

9.)  $\vec{PQ} = (2-1, -1-3) = (1, -4)$

10.)  $O = (0, 0)$ ,  $P = \left(\frac{2+(-4)}{2}, \frac{-1+3}{2}\right) = (-1, 1)$ , so

$\vec{OP} = (-1-0, 1-0) = (-1, 1)$

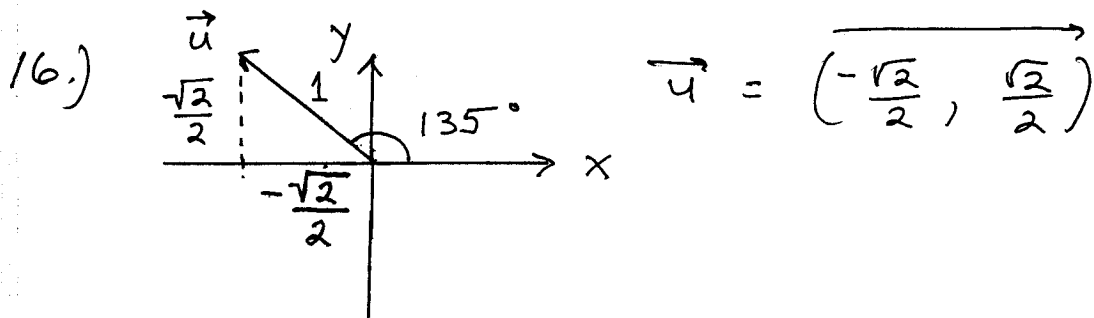
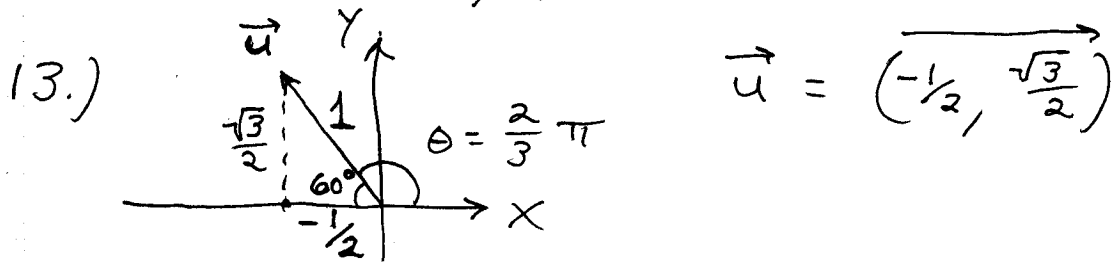
11.)  $A = (2, 3)$ ,  $O = (0, 0)$ , so

$\vec{AO} = (0-2, 0-3) = (-2, -3)$

$$12.) \vec{AB} = \overrightarrow{(2-1, 0-(-1))} = \overrightarrow{(1, 1)},$$

$$\vec{CD} = \overrightarrow{(-2-(-1), 2-3)} = \overrightarrow{(-1, -1)}, \text{ so}$$

$$\vec{AB} + \vec{CD} = \overrightarrow{(0, 0)}$$



$$18.) \vec{P_1 P_2} = \overrightarrow{(-3-1, 0-2, 5-0)} = \overrightarrow{(-4, -2, 5)}$$

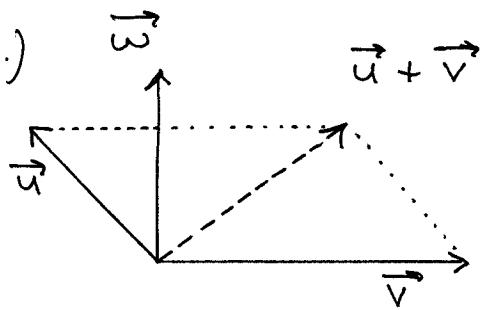
$$= -4\vec{i} - 2\vec{j} + 5\vec{k}$$

$$21.) 5\vec{u} - \vec{v} = 5\overrightarrow{(1, 1, -1)} - \overrightarrow{(2, 0, 3)}$$

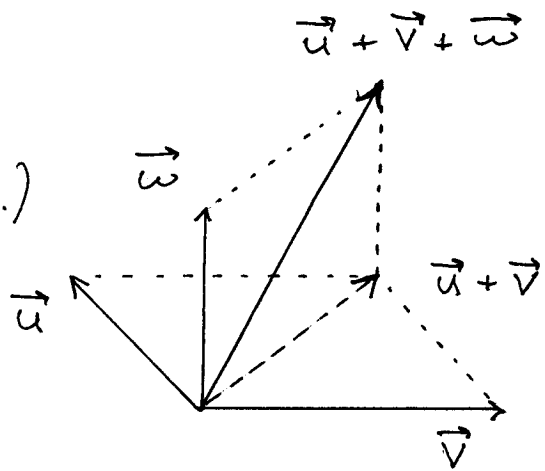
$$= \overrightarrow{(5, 5, -5)} - \overrightarrow{(2, 0, 3)} = \overrightarrow{(3, 5, -8)}$$

$$= 3\vec{i} + 5\vec{j} - 8\vec{k}$$

23.) a.)

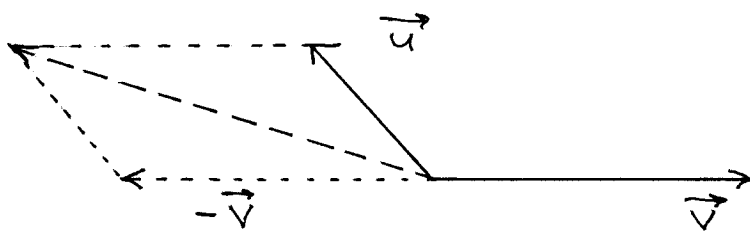


b.)

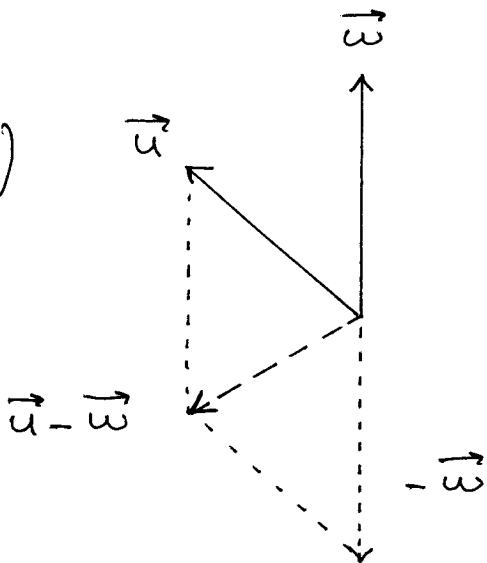


$\vec{u} - \vec{v}$

c.)



d.)



$$25.) \quad |2\vec{i} + \vec{j} - 2\vec{k}| = \sqrt{2^2 + 1^2 + (-2)^2}$$

$$= \sqrt{9} = 3, \text{ so}$$

$$2\vec{i} + \vec{j} - 2\vec{k} = 3 \cdot \frac{1}{3} (2\vec{i} + \vec{j} - 2\vec{k})$$

$$= 3 \cdot \left( \frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k} \right)$$

$$28.) \quad \left| \frac{3}{5}\vec{i} + \frac{4}{5}\vec{k} \right| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{25}{25}} = 1, \text{ so}$$

$$\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k} = 1 \cdot \left(\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}\right)$$

$$29.) \left| \frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k} \right| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2}$$

$$= \sqrt{\frac{1}{6} + \frac{1}{6} + \frac{1}{6}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}, \text{ so}$$

$$\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k}\right)$$

$$= \frac{1}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{6}}\vec{i} - \frac{\sqrt{2}}{\sqrt{6}}\vec{j} - \frac{\sqrt{2}}{\sqrt{6}}\vec{k}\right)$$

$$= \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)$$

$$31.) \text{ a.) } \vec{\omega} = 2 \cdot \vec{i}$$

$$\text{ b.) } \vec{\omega} = \sqrt{3} \cdot -\vec{k} = -\sqrt{3} \cdot \vec{k}$$

$$\text{ c.) } \vec{\omega} = \frac{1}{2} \cdot \left(\frac{3}{5}\vec{j} + \frac{4}{5}\vec{k}\right) = \frac{3}{10}\vec{j} + \frac{4}{10}\vec{k}$$

$$\text{ d.) } \vec{\omega} = 7 \cdot \left(\frac{6}{7}\vec{i} - \frac{2}{7}\vec{j} + \frac{3}{7}\vec{k}\right) = 6\vec{i} - 2\vec{j} + 3\vec{k}$$

$$33.) \vec{V} = 12\vec{i} - 5\vec{k} \rightarrow |\vec{V}| = \sqrt{12^2 + (-5)^2} = 13$$

so direction is  $\vec{u} = \frac{1}{13}\vec{V} = \frac{12}{13}\vec{i} - \frac{5}{13}\vec{k}$ ,  
so vector in same direction  
of magnitude 7 is

$$\vec{\omega} = 7\vec{u} = \frac{84}{13}\vec{i} - \frac{35}{13}\vec{k}$$

$$35.) \text{ a.) } \vec{P_1 P_2} = \overrightarrow{(2 - (-1), 5 - 1, 0 - 5)}$$

$$= \overrightarrow{(3, 4, -5)} \rightarrow |\overrightarrow{(3, 4, -5)}| = \sqrt{3^2 + 4^2 + (-5)^2}$$

$$= \sqrt{50} = 5\sqrt{2}, \text{ so direction is}$$

$$\vec{u} = \frac{1}{5\sqrt{2}} (3, 4, -5) = \left( \frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$$

b.) midpoint:  $\left( \frac{2-1}{2}, \frac{5+1}{2}, \frac{0+5}{2} \right) = \left( \frac{1}{2}, 3, \frac{5}{2} \right)$

38.) a.)  $\vec{P_1 P_2} = (2, -2, -2) \rightarrow$

$$|(2, -2, -2)| = \sqrt{2^2 + (-2)^2 + (-2)^2} = \sqrt{12} = 2\sqrt{3},$$

so direction is

$$\vec{u} = \frac{1}{2\sqrt{3}} (2, -2, -2) = \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)$$

b.) midpoint:  $\left( \frac{0+2}{2}, \frac{0-2}{2}, \frac{0-2}{2} \right) = (1, -1, -1)$

40.)  $\vec{AB} = -7\vec{i} + 3\vec{j} + 8\vec{k}$ ,  $A = (-2, -3, 6)$ ,

$B = (a, b, c)$ , so

$$(a - (-2), b - (-3), c - 6) = (a + 2, b + 3, c - 6) = (-7, 3, 8)$$

$$\begin{array}{l} \rightarrow a + 2 = -7 \rightarrow a = -9 \\ \rightarrow b + 3 = 3 \rightarrow b = 0 \\ \rightarrow c - 6 = 8 \rightarrow c = 14 \end{array} \left. \vphantom{\begin{array}{l} \rightarrow a + 2 = -7 \\ \rightarrow b + 3 = 3 \\ \rightarrow c - 6 = 8 \end{array}} \right\} \text{ so}$$

$$B = -9\vec{i} + 0\vec{j} + 14\vec{k} = -9\vec{i} + 14\vec{k}$$

41.)  $\vec{u} = 2\vec{i} + \vec{j}$ ,  $\vec{v} = \vec{i} + \vec{j}$ , and

$\vec{w} = \vec{i} - \vec{j}$ ; we want

$$\vec{u} = a\vec{v} + b\vec{w} \rightarrow$$

$$(2, 1) = a(1, 1) + b(1, -1) \rightarrow$$

$$\overrightarrow{(2, 1)} = \overrightarrow{(a+b, a-b)} \rightarrow$$

$$\left. \begin{array}{l} a+b=2 \\ a-b=1 \end{array} \right\} 2a=3 \rightarrow a = \frac{3}{2}$$

$$\frac{3}{2} + b = 2 \rightarrow b = 2 - \frac{3}{2} \rightarrow b = \frac{1}{2}$$