

Section 12.3

$$1.) \vec{v} = (2, -4, \sqrt{5}), \vec{u} = (-2, 4, -\sqrt{5})$$

$$a.) \vec{v} \cdot \vec{u} = (-4) + (-16) + (-5) = -25,$$

$$|\vec{u}| = \sqrt{4+16+5} = \sqrt{25} = 5,$$

$$|\vec{v}| = \sqrt{4+16+5} = \sqrt{25} = 5.$$

$$b.) \cos \theta = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = \frac{-25}{(5)(5)} = -1 \quad (\rightarrow \theta = \pi)$$

$$c.) |\vec{u}| \cos \theta = (5)(-1) = -5$$

$$d.) \text{proj}_{\vec{v}} \vec{u} = \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \frac{-25}{(5)^2} \vec{v}$$

$$= -\vec{v} = (-2, 4, -\sqrt{5})$$

$$4.) \vec{v} = (2, 10, -11), \vec{u} = (2, 2, 1)$$

$$a.) \vec{v} \cdot \vec{u} = 4 + 20 + (-11) = 13,$$

$$|\vec{u}| = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$|\vec{v}| = \sqrt{4+100+121} = \sqrt{225} = 15$$

$$b.) \cos \theta = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = \frac{13}{(15)(3)} = \frac{13}{45}$$

$$c.) |\vec{u}| \cos \theta = (3) \cdot \left( \frac{13}{45} \right) = \frac{13}{15}$$

$$d.) \text{proj}_{\vec{v}} \vec{u} = \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \frac{13}{(15)^2} \vec{v}$$

$$= \frac{13}{225} (2, 10, -11) = \left( \frac{26}{225}, \frac{130}{225}, \frac{-143}{225} \right)$$

$$7.) \vec{v} = \overrightarrow{(5, 1)}, \quad \vec{u} = \overrightarrow{(2, \sqrt{17})}$$

$$a.) \vec{v} \cdot \vec{u} = 10 + \sqrt{17}$$

$$|\vec{u}| = \sqrt{4 + 17} = \sqrt{21}$$

$$|\vec{v}| = \sqrt{25 + 1} = \sqrt{26}$$

$$b.) \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{10 + \sqrt{17}}{\sqrt{21} \sqrt{26}} = \frac{10 + \sqrt{17}}{\sqrt{546}}$$

$$c.) |\vec{u}| \cos \theta = \cancel{\sqrt{21}} \cdot \frac{10 + \sqrt{17}}{\sqrt{21} \sqrt{26}} = \frac{10 + \sqrt{17}}{\sqrt{26}}$$

$$d.) \text{proj}_{\vec{v}} \vec{u} = \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

$$= \frac{10 + \sqrt{17}}{26} \vec{v} = \left( \frac{5(10 + \sqrt{17})}{26}, \frac{10 + \sqrt{17}}{26} \right)$$

$$10.) \vec{u} = \overrightarrow{(2, -2, 1)}, \quad \vec{v} = \overrightarrow{(3, 0, 4)}, \quad \text{so}$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{6 + 0 + 4}{\sqrt{9} \cdot \sqrt{25}} = \frac{10}{(3)(5)} = \frac{2}{3} \rightarrow$$

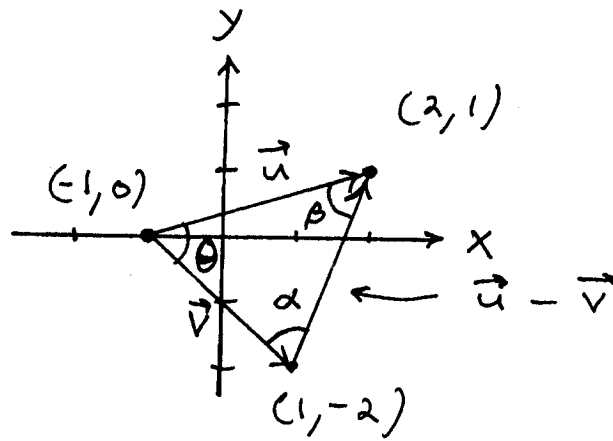
$$\theta = \arccos \frac{2}{3} \approx 0.84 \text{ radians}$$

$$12.) \vec{u} = \overrightarrow{(1, \sqrt{2}, -\sqrt{2})}, \quad \vec{v} = \overrightarrow{(-1, 1, 1)}, \quad \text{so}$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{-1 + \sqrt{2} - \sqrt{2}}{\sqrt{5} \sqrt{3}} = \frac{-1}{\sqrt{15}} \rightarrow$$

$$\theta = \arccos \left( \frac{-1}{\sqrt{15}} \right) \approx 1.83 \text{ radians}$$

13.)



$$\vec{u} = (3, 1), \quad \vec{v} = (2, -2), \quad \vec{u} - \vec{v} = (1, 3);$$

angle between  $\vec{u}$  and  $\vec{v}$  satisfies

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{6 - 2}{\sqrt{10} \cdot \sqrt{8}} = \frac{4}{\sqrt{80}} = \frac{1}{\sqrt{5}} \rightarrow$$

$$\theta = \arccos \frac{1}{\sqrt{5}} \approx 63.4^\circ;$$

angle between  $-\vec{v}$  and  $\vec{u} - \vec{v}$  satisfies

$$\cos \alpha = \frac{-\vec{v} \cdot (\vec{u} - \vec{v})}{|-\vec{v}| |\vec{u} - \vec{v}|} = \frac{(-2, 2) \cdot (1, 3)}{|(-2, 2)| |(1, 3)|}$$

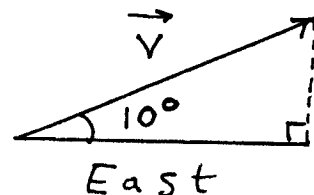
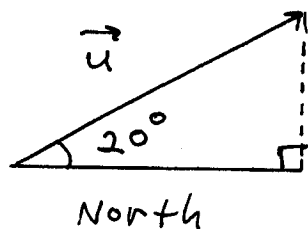
$$= \frac{-2 + 6}{\sqrt{8} \sqrt{10}} = \frac{4}{\sqrt{80}} = \frac{1}{\sqrt{5}} \rightarrow$$

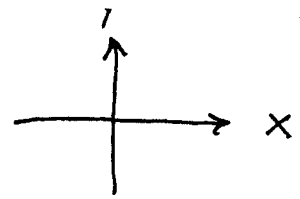
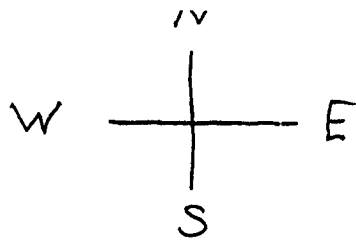
$$\alpha = \arccos \frac{1}{\sqrt{5}} \approx 63.4^\circ;$$

$$\theta + \alpha + \beta = 180^\circ \rightarrow$$

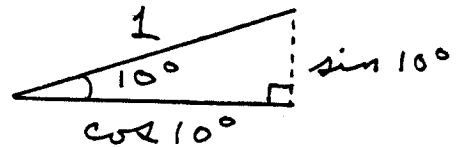
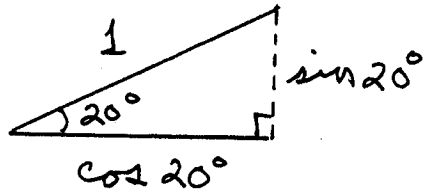
$$\beta = 180^\circ - \alpha - \theta \approx 53.2^\circ$$

16.)





Assume  $\vec{u}$  and  $\vec{v}$  are unit vectors in space. Then



so that

$$\vec{u} = 0\vec{i} + (\cos 20^\circ)\vec{j} + (\sin 20^\circ)\vec{k}$$

and  $\vec{v} = (\cos 10^\circ)\vec{i} + 0\vec{j} + (\sin 10^\circ)\vec{k}$  ;

the angle  $\theta$  in the water main is the angle between  $-\vec{u}$  and  $\vec{v}$

(since tails must be together !!)

$$\text{so } \cos \theta = \frac{(-\vec{u}) \cdot \vec{v}}{|-\vec{u}| |\vec{v}|} = \frac{0 + 0 - (\sin 20^\circ)(\sin 10^\circ)}{(1)(1)} \rightarrow$$

$$\theta = \arccos(-\sin 20^\circ \cdot \sin 10^\circ) \approx 93.4^\circ$$

17) Vectors  $\vec{v}_1 + \vec{v}_2$  and  $\vec{v}_1 - \vec{v}_2$  are  $\perp$

iff  $(\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2) = 0$

iff  $(\vec{v}_1 + \vec{v}_2) \cdot \vec{v}_1 - (\vec{v}_1 + \vec{v}_2) \cdot \vec{v}_2 = 0$

(by property 3)

$$\text{iff } \vec{v}_1 \cdot \vec{v}_1 + \vec{v}_2 \cdot \vec{v}_1 - \vec{v}_1 \cdot \vec{v}_2 - \vec{v}_2 \cdot \vec{v}_2 = 0$$

(by property 3)

$$\text{iff } |\vec{v}_1|^2 + \cancel{\vec{v}_1 \cdot \vec{v}_2} - \cancel{\vec{v}_1 \cdot \vec{v}_2} - |\vec{v}_2|^2 = 0$$

(by properties 1 and 4)

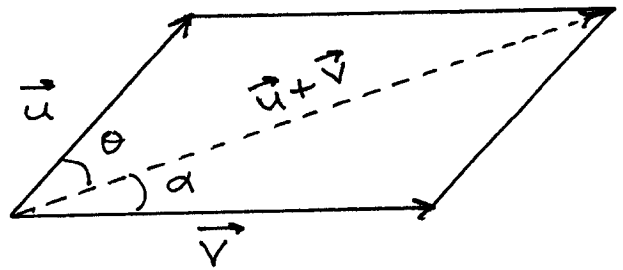
$$\text{iff } |\vec{v}_1|^2 = |\vec{v}_2|^2$$

$$\text{iff } |\vec{v}_1| = |\vec{v}_2|, \text{ i.e.,}$$

vectors  $\vec{v}_1$  and  $\vec{v}_2$  have the same magnitude.

22) Show

that angles  $\theta = \alpha$  if  $|\vec{u}| = |\vec{v}|$ :



$$\cos \theta = \frac{\vec{u} \cdot (\vec{u} + \vec{v})}{|\vec{u}| |\vec{u} + \vec{v}|} = \frac{\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{u} + \vec{v}|}$$

$$= \frac{|\vec{u}|^2 + \vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{u} + \vec{v}|}$$

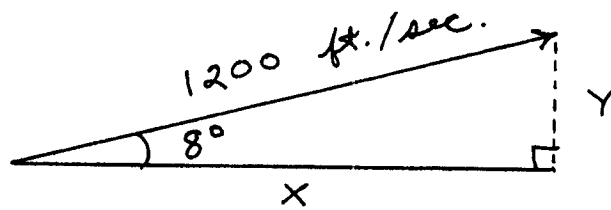
$$\cos \alpha = \frac{\vec{v} \cdot (\vec{u} + \vec{v})}{|\vec{v}| |\vec{u} + \vec{v}|} = \frac{\vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}}{|\vec{v}| |\vec{u} + \vec{v}|}$$

$$= \frac{\vec{u} \cdot \vec{v} + |\vec{v}|^2}{|\vec{v}| |\vec{u} + \vec{v}|} = \frac{\vec{u} \cdot \vec{v} + |\vec{u}|^2}{|\vec{u}| |\vec{u} + \vec{v}|}$$

(since  $|\vec{u}| = |\vec{v}|$ ) ;

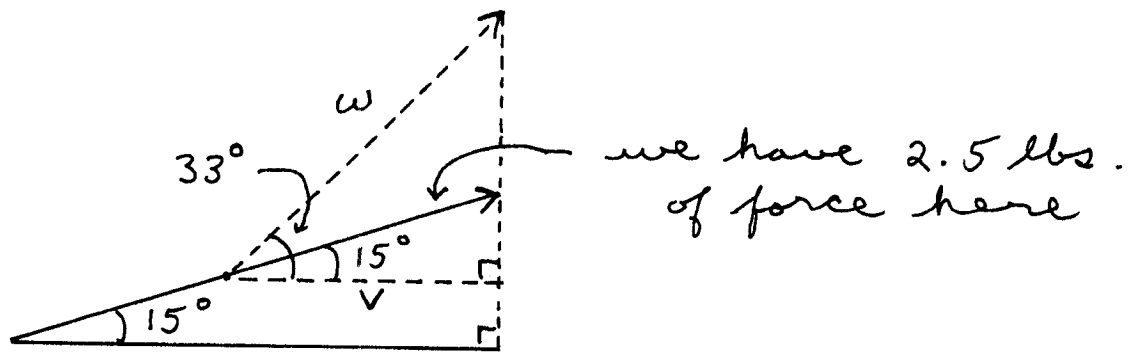
then  $\cos \theta = \cos \alpha \rightarrow \theta = \alpha$ .

23)



$$\cos 8^\circ = \frac{x}{1200} \rightarrow x = 1200 \cos 8^\circ \approx 1188.32 \text{ ft./sec.}$$
$$\sin 8^\circ = \frac{y}{1200} \rightarrow y = 1200 \sin 8^\circ \approx 167.01 \text{ ft./sec.}$$

24)



$$\cos 15^\circ = \frac{v}{2.5} \rightarrow v = 2.5 \cos 15^\circ$$

$$\cos 33^\circ = \frac{v}{w} \rightarrow$$

$$w = \frac{v}{\cos 33^\circ} = \frac{2.5 \cos 15^\circ}{\cos 33^\circ} \approx 2.88 \text{ lbs.}$$

25)

a.)  $\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta \rightarrow$

$$|\vec{u} \cdot \vec{v}| = ||\vec{u}| |\vec{v}| \cos \theta|$$

$$= |\vec{u}| |\vec{v}| \cdot |\cos \theta|$$

$$\leq |\vec{u}| |\vec{v}| \cdot (1) \rightarrow$$

$$|\vec{u} \cdot \vec{v}| \leq |\vec{u}| |\vec{v}|$$

b.) If  $|\cos\theta| = 1$ , then

$$|\vec{u} \cdot \vec{v}| = |\vec{u}| |\vec{v}| \cdot |\cos\theta|$$

$$= |\vec{u}| |\vec{v}| \cdot (1)$$

$$= |\vec{u}| |\vec{v}|$$

This equality holds if  $\theta = 0^\circ$  (vectors have same direction) or if  $\theta = 180^\circ$  (vectors have opposite direction)

27) assume  $\vec{u}_1$  is  $\perp$  to  $\vec{u}_2$  and  $|\vec{u}_1| = 1 = |\vec{u}_2|$ . If  $\vec{v} = a\vec{u}_1 + b\vec{u}_2$ , then

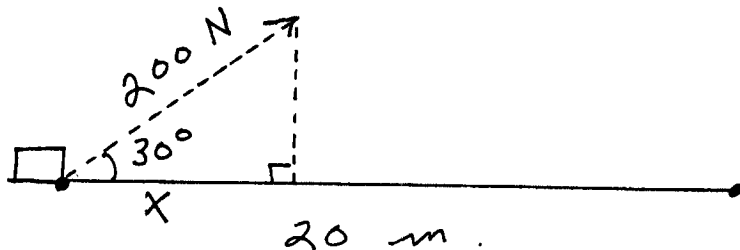
$$\vec{v} \cdot \vec{u}_1 = (a\vec{u}_1 + b\vec{u}_2) \cdot \vec{u}_1$$

$$= a(\vec{u}_1 \cdot \vec{u}_1) + b(\vec{u}_2 \cdot \vec{u}_1)$$

$$= a|\vec{u}_1|^2 + b \cdot (0) \quad (\text{since } \vec{u}_2 \perp \vec{u}_1)$$

$$= a(1)^2 = a$$

43)



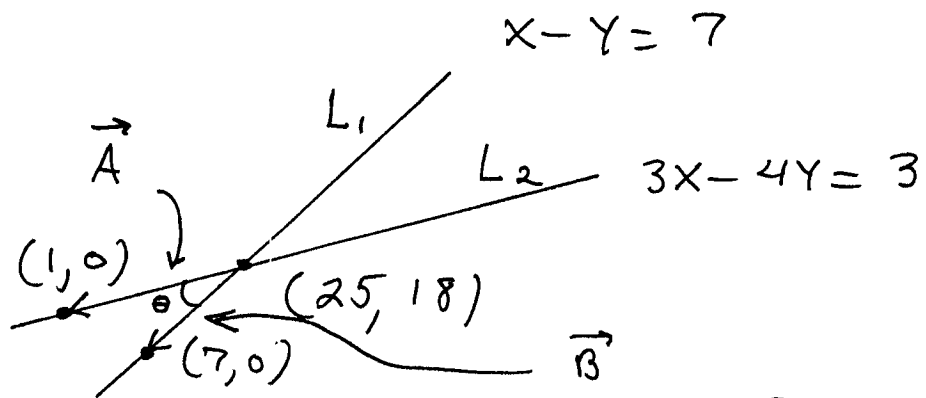
$$\cos 30^\circ = \frac{x}{200} \rightarrow x = 200 \cos 30^\circ$$

$$= 200 \left(\frac{\sqrt{3}}{2}\right) = 100\sqrt{3} \text{ N}$$

$$\text{Work} = (\text{force})(\text{distance})$$

$$= (100\sqrt{3})(20) \approx 3464 \text{ joules}$$

49)



$$\left. \begin{array}{l} 3x - 4y = 3 \\ x - y = 7 \end{array} \right\} \quad \left. \begin{array}{l} 3x - 4y = 3 \\ -4x + 4y = -28 \end{array} \right\}$$

$-x = -25 \rightarrow x = 25, y = 18$  is  
 pt. of intersection ; pt.  $(7, 0)$  is  
 on line  $L_1$ , and pt.  $(1, 0)$  is on line  $L_2$  ;  
 form vectors  $\vec{A}$  and  $\vec{B}$  and find  
 angle  $\theta$  between them :

$$\vec{A} = \overrightarrow{(-24, -18)} \quad \text{and} \quad \vec{B} = \overrightarrow{(-18, -18)}, \text{ so}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{432 + 324}{\sqrt{900} \cdot \sqrt{648}} = \frac{756}{30\sqrt{648}} \rightarrow$$

$$\theta = \arccos \left( \frac{756}{30\sqrt{648}} \right) \approx 8.13^\circ$$