Section 12.4

2.) $\vec{u} = \overrightarrow{(2,3,0)}$, $\vec{v} = \overrightarrow{(-1,1,0)}$ then

a.) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} \vec{k}$

$= (0)\vec{i} - (0)\vec{j} + 5\vec{k} = 5\vec{k}$;

$|\vec{u} \times \vec{v}| = 5$ and direction of $\vec{u} \times \vec{v}$ is $\vec{k}$.

b.) $\vec{v} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 2 & 3 & 0 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} \vec{k}$

$= (0)\vec{i} - (0)\vec{j} + (-5)\vec{k} = -5\vec{k}$;

$|\vec{v} \times \vec{u}| = 5$ and direction of $\vec{v} \times \vec{u}$ is $-\vec{k}$

3.) $\vec{u} = (2,-2,4)$, $\vec{v} = (-1,1,-2)$ then

a.) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 4 \\ -1 & 1 & -2 \end{vmatrix}$

$= \begin{vmatrix} -2 & 4 \\ 1 & -2 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix} \vec{k}$

$= (0)\vec{i} - (0)\vec{j} + (0)\vec{k} = \overrightarrow{(0,0,0)}$;

$|\vec{u} \times \vec{v}| = 0$ and $\vec{u} \times \vec{v}$ has no direction

b.) $\vec{v} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{vmatrix}$

$$= \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} -1 & 1 \\ 2 & -2 \end{vmatrix} \vec{k}$$

$$= (0)\vec{i} - (0)\vec{j} + (0)\vec{k} = \overrightarrow{(0,0,0)} ;$$

$|\vec{v} \times \vec{u}| = 0$ and $\vec{v} \times \vec{u}$ has no direction.

7.) $\vec{u} = \overrightarrow{(-8,-2,-4)}$, $\vec{v} = \overrightarrow{(2,2,1)}$ then

a.) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & -2 & -4 \\ 2 & 2 & 1 \end{vmatrix}$

$$= \begin{vmatrix} -2 & -4 \\ 2 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} -8 & -4 \\ 2 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} -8 & -2 \\ 2 & 2 \end{vmatrix} \vec{k}$$

$$= 6\vec{i} - (0)\vec{j} + (-12)\vec{k} = \overrightarrow{(6,0,-12)} ;$$

$|\vec{u} \times \vec{v}| = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$ and direction of $\vec{u} \times \vec{v}$ is $\frac{1}{6\sqrt{5}} \overrightarrow{(6,0,-12)} = \overrightarrow{\left(\frac{1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}\right)}$.

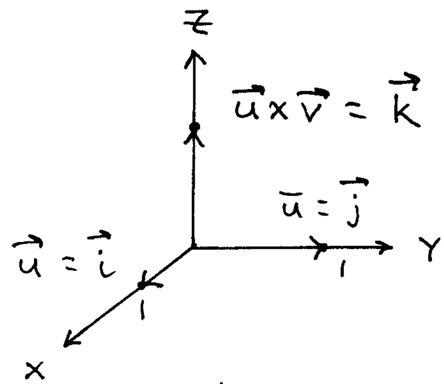
b.) $\vec{v} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ -8 & -2 & -4 \end{vmatrix}$

$$= \begin{vmatrix} 2 & 1 \\ -2 & -4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 1 \\ -8 & -4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 2 \\ -8 & -2 \end{vmatrix} \vec{k}$$

$$= (-6)\vec{i} - (0)\vec{j} + 12\vec{k} = \overrightarrow{(-6,0,12)} ;$$

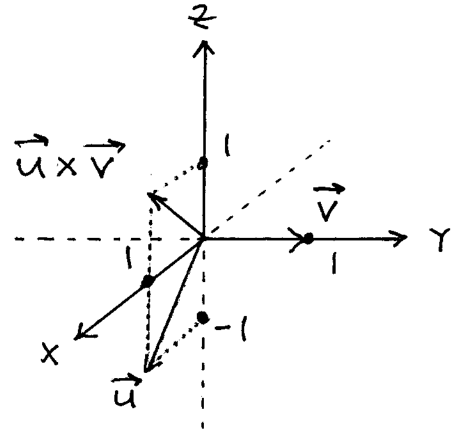
$|\vec{v} \times \vec{u}| = \sqrt{180} = 6\sqrt{5}$ and direction of $\vec{v} \times \vec{u}$ is $\frac{1}{6\sqrt{5}} \overrightarrow{(-6,0,12)} = \overrightarrow{\left(\frac{-1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)}$.

9.)



$$\vec u \times \vec V = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \vec i - \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \vec j + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \vec k$$
$$= (0)\vec i - (0)j + (1)\vec k = k$$

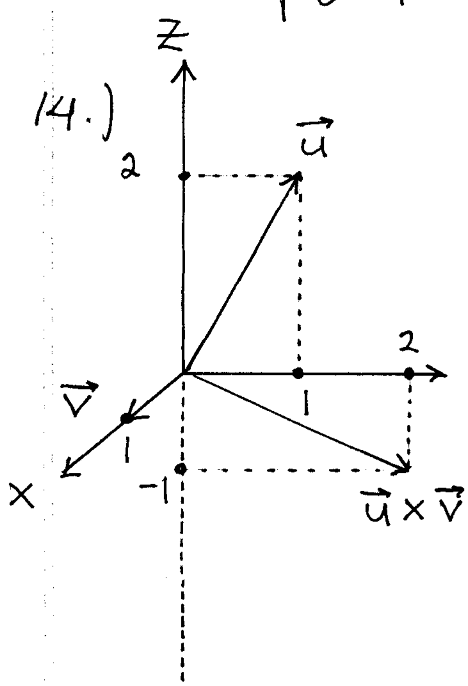
10.)



$\vec u = \vec i - \vec k = \overrightarrow{(1,0,-1)}$, $\quad \vec V = \vec j = \overrightarrow{(0,1,0)}$ then

$$\vec u \times \vec V = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \vec i - \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} \vec j + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \vec k$$
$$= (1)\vec i - (0)\vec j + (1)\vec k = \vec i + \vec k$$

14.)



$\vec u = \vec j + 2\vec k = \overrightarrow{(0,1,2)}$,

$\vec V = \vec i = \overrightarrow{(1,0,0)}$ then

$$\vec u \times \vec V = \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} \vec i - \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} \vec j + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \vec k$$
$$= (0)\vec i - (-2)\vec j + (-1)\vec k$$
$$= 2\vec j - \vec k$$

16.) $P = (2,-2,1)$, $Q = (2,1,3)$, $R = (3,-1,1)$ so
$\overrightarrow{PQ} = \overrightarrow{(0,3,2)}$ and $\overrightarrow{PR} = \overrightarrow{(1,1,0)}$;

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 2 \\ 1 & 1 & 0 \end{vmatrix} = (-2)\vec{i} - (-2)\vec{j} + (-3)\vec{k}$$
$$= -2\vec{i} + 2\vec{j} - 3\vec{k} \; ;$$

area of Δ is $\frac{1}{2}$ area of ▱ so

a.) Area $= \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right|$
$= \frac{1}{2}\sqrt{4+4+9} = \frac{1}{2}\sqrt{17}$;

b.) a unit vector $\perp$ to plane PQR is

$$\vec{u} = \frac{1}{\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|} \cdot \overrightarrow{PQ} \times \overrightarrow{PR}$$
$$= \frac{1}{17}\overrightarrow{(-2,2,-3)} = \overrightarrow{\left(\frac{-2}{17}, \frac{2}{17}, \frac{-3}{17}\right)}$$

19.) $\vec{u} = 2\vec{i}$, $\vec{v} = 2\vec{j}$, $\vec{w} = 2\vec{k}$ then

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = (0)\vec{i} - (0)\vec{j} + 4\vec{k} = 4\vec{k} \; ,$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 4\vec{i} - (0)\vec{j} + (0)\vec{k} = 4\vec{i} \; ,$$

$$\vec{w} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 2 \\ 2 & 0 & 0 \end{vmatrix} = (0)\vec{i} - (-4)\vec{j} + (0)\vec{k}$$
$$= 4\vec{j} \; ;$$

$(\vec{u} \times \vec{v}) \cdot \vec{w} = \overrightarrow{(0,0,4)} \cdot \overrightarrow{(0,0,2)} = 0+0+8 = 8,$

$(\vec{v} \times \vec{w}) \cdot \vec{u} = \overrightarrow{(4,0,0)} \cdot \overrightarrow{(2,0,0)} = 8+0+0 = 8,$

$(\vec{w} \times \vec{u}) \cdot \vec{v} = \overrightarrow{(0,4,0)} \cdot \overrightarrow{(0,2,0)} = 0+8+0 = 8;$

volume of parallelepiped is

$\text{Volume} = |(\vec{u} \times \vec{v}) \cdot \vec{w}| = |8| = 8$

20.) $\vec{u} = \overrightarrow{(1,-1,1)}$, $\vec{v} = \overrightarrow{(2,1,-2)}$, $\vec{w} = \overrightarrow{(-1,2,-1)}$ then

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = (1)\vec{i} - (-4)\vec{j} + (3)\vec{k} = \vec{i} + 4\vec{j} + 3\vec{k},$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix} = (3)\vec{i} - (-4)\vec{j} + (5)\vec{k} = 3\vec{i} + 4\vec{j} + 5\vec{k},$$

$$\vec{w} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = (1)\vec{i} - (0)\vec{j} + (-1)\vec{k} = \vec{i} - \vec{k};$$

$(\vec{u} \times \vec{v}) \cdot \vec{w} = \overrightarrow{(1,4,3)} \cdot \overrightarrow{(-1,2,-1)} = -1+8-3 = 4,$

$(\vec{v} \times \vec{w}) \cdot \vec{u} = \overrightarrow{(3,4,5)} \cdot \overrightarrow{(1,-1,1)} = 3-4+5 = 4,$

$(\vec{w} \times \vec{u}) \cdot \vec{v} = \overrightarrow{(1,0,-1)} \cdot \overrightarrow{(2,1,-2)} = 2+0+2 = 4;$

volume of parallelepiped is

$\text{Volume} = |(\vec{u} \times \vec{v}) \cdot \vec{w}| = |4| = 4$

23.) $\vec{u} = 5\vec{i} - \vec{j} + \vec{k}$, $\vec{v} = \vec{j} - 5\vec{k}$,

$\vec{w} = -15\vec{i} + 3\vec{j} - 3\vec{k} = -3(5\vec{i} - \vec{j} + \vec{k}) = -3\vec{u}$:

a.) $\vec{u}\cdot\vec{v} = (5)(0)+(-1)(1)+(1)(-5) = -6$,

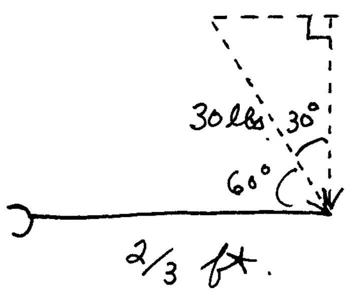
$\vec{v}\cdot\vec{w} = \vec{v}\cdot(-3\vec{u}) = -3(\vec{u}\cdot\vec{v}) = -3(-6) = 18$,

$\vec{u}\cdot\vec{w} = \vec{u}\cdot(-3\vec{u}) = -3\left((5)^2+(-1)^2+(1)^2\right) = -81$;

so no vectors are $\perp$

b.) $\vec{w} = -3\vec{u}$ so $\vec{u}$ and $\vec{w}$ are $\parallel$.

25.)



$\frac{\sqrt{3}}{2}\cdot(30) = 15\sqrt{3}$ lbs.

Torque = (vert. force)(distance)
$= (15\sqrt{3})(2/3) = 10\sqrt{3}$ ft.-lbs.

27.)
a.) True
b.) False
c.) True
d.) True
e.) False
f.) True
g.) True
h.) True

28.)
a.) True
b.) True
c.) True
d.) True
e.) True
f.) True
g.) True
h.) True

29.) a.) $\text{proj.}_{\vec{v}}\,\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$

b.) $\vec{u}\times\vec{v}$ is $\perp$ to $\vec{u}$ and $\vec{v}$

c.) $(\vec{u}\times\vec{v})\times\vec{w}$ is $\perp$ to $\vec{u}\times\vec{v}$ and $\vec{w}$

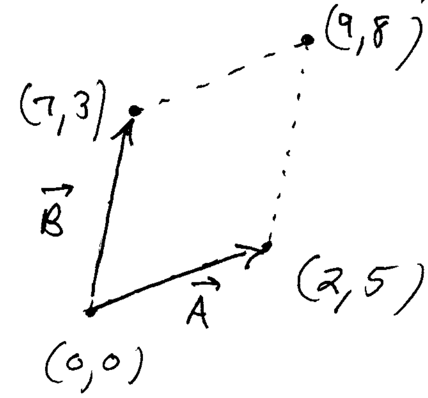
d.) Volume of parallelepiped is $\left|(\vec{u}\times\vec{v})\cdot\vec{w}\right|$

31.) a.) Sense
b.) Not
c.) Sense
d.) Not

33.) Not necessarily : Let $\vec{v} = \vec{0}$ and $\vec{w} = 2\vec{u}$. Then

$$\vec{u} \times \vec{v} = \vec{u} \times \vec{0} = \vec{0}$$ and
$$\vec{u} \times \vec{w} = \vec{u} \times (2\vec{u}) = 2(\vec{u} \times \vec{u}) = 2(\vec{0}) = \vec{0},$$
but $\vec{v} \neq \vec{w}$.

36.) $\vec{A} = \overrightarrow{(2,5,0)}$, $\vec{B} = \overrightarrow{(7,3,0)}$
so area of parallelogram is $|\vec{A} \times \vec{B}|$:

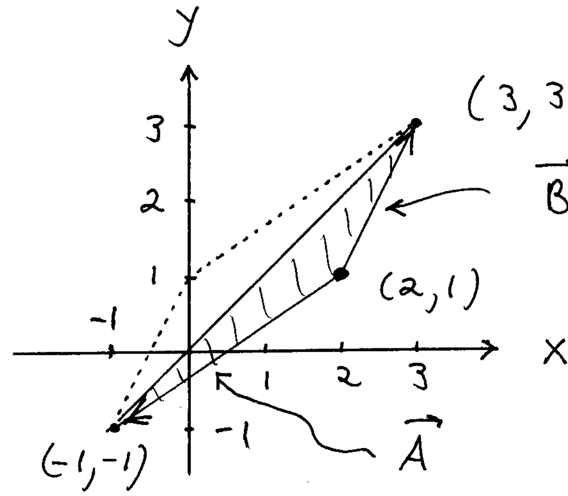


$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 0 \\ 7 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 5 & 0 \\ 3 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ 7 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 5 \\ 7 & 3 \end{vmatrix} \vec{k}$$

$$= (0-0)\vec{i} - (0-0)\vec{j} + (-29)\vec{k} = -29\vec{k},$$

so Area $= |\vec{A} \times \vec{B}| = |-29\vec{k}| = 29$

40.)



area of triangle is $\frac{1}{2}$ area of parallelogram ; so

$\vec{A} = \overrightarrow{(-3,-2,0)}$, $\vec{B} = \overrightarrow{(1,2,0)}$,

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -2 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 0 \\ 2 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} -3 & 0 \\ 1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} -3 & -2 \\ 1 & 2 \end{vmatrix} \vec{k}$$

$$= (0-0)\vec{i} - (0-0)\vec{j} + (-4)\vec{k} = -4\vec{k} \; ;$$

$$\text{Area of } \Delta = \frac{1}{2} |\vec{A} \times \vec{B}|$$

$$= \frac{1}{2} |-4\vec{k}|$$

$$= \frac{1}{2}(4)$$

$$= 2 \; .$$