Section 12.5

1.) point $P = (3, -4, -1)$, vector $\vec{A} = \overrightarrow{(1,1,1)}$, so

line $L: \begin{cases} x = 3 + (1)t = 3 + t \\ y = -4 + (1)t = -4 + t \\ z = -1 + (1)t = -1 + t \end{cases}$

4.) points $P = (1, 2, 0)$, $Q = (1, 1, -1)$ and vector $\vec{PQ} = \overrightarrow{(0,-1,-1)}$, so line

$$L: \begin{cases} x = 1 + (0)t = 1 \\ y = 2 + (-1)t = 2 - t \\ z = 0 + (-1)t = -t \end{cases}$$

6.) point $P = (3, -2, 1)$ and $\|$ to line

$L: \begin{cases} x = 1 + 2t \\ y = 2 - t \\ z = 3t \end{cases}$ so $\|$ vector is $\vec{A} = \overrightarrow{(2,-1,3)}$

and line is

$$M: \begin{cases} x = 3 + (2)t = 3 + 2t \\ y = -2 + (-1)t = -2 - t \\ z = 1 + (3)t = 1 + 3t \end{cases}$$

7.) point $P = (1, 1, 1)$ and $\|$ to z-axis so $\|$ vector is $\vec{A} = \overrightarrow{(0,0,1)}$, and line is

$$L: \begin{cases} x = 1 + (0)t = 1 \\ y = 1 + (0)t = 1 \\ z = 1 + (1)t = 1 + t \end{cases}$$

8.) point $P = (2, 4, 5)$ and $\perp$ to plane

$3X + 7Y - 5Z = 21$; plane has $\perp$ vector $\vec{A} = \overrightarrow{(3, 7, -5)}$, so line is

$$L: \begin{cases} x = 2 + (3)t = 2 + 3t \\ y = 4 + (7)t = 4 + 7t \\ z = 5 + (-5)t = 5 - 5t \end{cases}$$

10.) $\vec{u} = \overrightarrow{(1, 2, 3)}$, $\vec{V} = \overrightarrow{(3, 4, 5)}$ so

$$\vec{u} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = (10-12)\vec{i} - (5-9)\vec{j} + (4-6)\vec{k}$$

$= -2\vec{i} + 4\vec{j} - 2\vec{k}$ is $\perp$ to $\vec{u}$ and $\vec{v}$, and point $P = (2, 3, 0)$ so line

$$L: \begin{cases} X = 2 + (-2)t = 2 - 2t \\ Y = 3 + (4)t = 3 + 4t \\ Z = 0 + (-2)t = -2t \end{cases}$$

21.) point $P = (0, 2, -1)$ and $\perp$ vector $\vec{u} = \overrightarrow{(3, -2, -1)}$, so plane is

$3(x - 0) - 2(Y - 2) - 1(Z - (-1)) = 0 \rightarrow$

$3X - 2Y + 4 - Z - 1 = 0 \rightarrow$

$3X - 2Y - Z = -3$.

22.) plane $3X + Y + Z = 7$ has $\perp$ vector $\vec{u} = \overrightarrow{(3, 1, 1)}$, and point $P = (1, -1, 3)$, so new plane is

$3(x - 1) + 1 \cdot (Y - (-1)) + 1 \cdot (Z - 3) = 0 \rightarrow$

$3x - 3 + y + 1 + z - 3 = 0 \rightarrow$

$3x + y + z = 5$

24.) points $P = (2, 4, 5)$, $Q = (1, 5, 7)$, $R = (-1, 6, 8)$ so vectors $\vec{PQ} = \overrightarrow{(-1, 1, 2)}$ and $\vec{PR} = \overrightarrow{(-3, 2, 3)}$, so $\perp$ vector to plane is

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix} = (3-4)\,\vec{i} - (-3+6)\,\vec{j} + (-2+3)\,\vec{k}$$

$= -\vec{i} - 3\,\vec{j} + \vec{k}$; equation of plane is $-1(x-2) - 3(y-4) + 1(z-5) = 0 \rightarrow$

$-x + 2 - 3y + 12 + z - 5 = 0 \rightarrow$

$-x - 3y + z = -9$

25.) Line $L: \begin{cases} x = 5 + t \\ y = 1 + 3t \\ z = 4t \end{cases}$ has $\parallel$ vector

$\vec{A} = \overrightarrow{(1, 3, 4)}$, so plane has $\perp$ vector $\vec{A} = \overrightarrow{(1, 3, 4)}$, so plane through point $P = (2, 4, 5)$ is

$1 \cdot (x-2) + 3 \cdot (y-4) + 4 \cdot (z-5) = 0 \rightarrow$

$x - 2 + 3y - 12 + 4z - 20 = 0 \rightarrow$

$x + 3y + 4z = 34$

28.) Lines $L_1: \begin{cases} x = t \\ y = 2 - t \\ z = 1 + t \end{cases}$ and $L_2: \begin{cases} x = 2 + 2s \\ y = 3 + s \\ z = 6 + 5s \end{cases}$;

if lines intersect, then

$\left.\begin{array}{l} t = 2 + 2s \\ 2 - t = 3 + s \end{array}\right\}$ (add) $\rightarrow 2 = 5 + 3s \rightarrow$

$s = -1$ and $t = 0$, so pt. of $\cap$ is $\boxed{P = (x, y, z) = (0, 2, 1)}$; vector $\parallel$ to L_1 is $\vec{A} = \overrightarrow{(1, -1, 1)}$, vector $\parallel$ to L_2 is $\vec{B} = \overrightarrow{(2, 1, 5)}$, so vector $\perp$ to plane is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = (-5-1)\vec{i} - (5-2)\vec{j} + (1+2)\vec{k}$$

$= \underline{\underline{-6\vec{i} - 3\vec{j} + 3\vec{k}}}$; then equation of plane is

$-6 \cdot (x - 0) - 3(y - 2) + 3(z - 1) = 0 \rightarrow$

$-6x - 3y + 6 + 3z - 3 = 0 \rightarrow$

$-6x - 3y + 3z = -3 \rightarrow$

$2x + y - z = 1$.

31.) plane $2x + y - z = 3$ has $\perp$ vector $\vec{A} = \overrightarrow{(2, 1, -1)}$; plane $x + 2y + z = 2$ has $\perp$ vector $\vec{B} = \overrightarrow{(1, 2, 1)}$, so plane $\parallel$ to the line of intersection of these planes is

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (1+2)\,\vec{i} - (2+1)\,\vec{j} + (4-1)\,\vec{k}$$

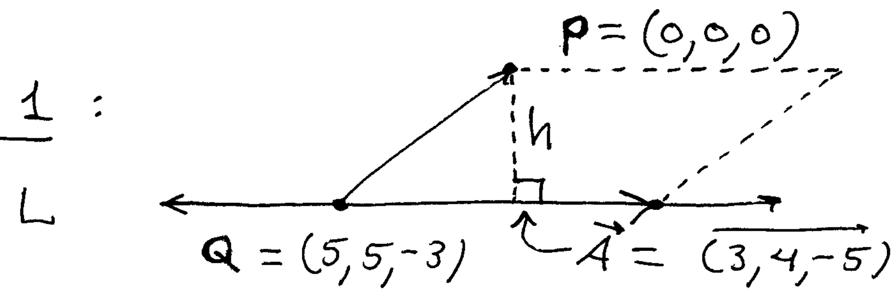
$$= \underline{\underline{3\vec{i} - 3\vec{j} + 3\vec{k}}}$$; so plane $\perp$ to $\vec{A}\times\vec{B}$ and through point $P = (2,1,-1)$ is

$$3(x-2) - 3(y-1) + 3(z+1) = 0 \rightarrow$$
$$(x-2) - (y-1) + (z+1) = 0 \rightarrow$$
$$x - y + z = 0$$

34.) point $P = (0,0,0)$, line

$$L: \begin{cases} x = 5+3t \\ y = 5+4t \\ z = -3-5t \end{cases}$$; distance from point P to line L is :

<ins>Method 1</ins> :



Point $Q = (5,5,-3)$ is on line L and vector $\vec{A} = \overrightarrow{(3,4,-5)}$ is $\parallel$ to L ; vector $\overrightarrow{QP} = \overrightarrow{(-5,-5,3)}$; area of parallelogram formed by $\overrightarrow{QP}$ and $\vec{A}$ is

$$|\overrightarrow{QP}\times\vec{A}| = (\text{base})(\text{height})$$
$$= |\vec{A}| \cdot h \Rightarrow$$

$$h = \frac{|\vec{QP} \times \vec{A}|}{|\vec{A}|} \quad ; \quad \vec{QP} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -5 & 3 \\ 3 & 4 & -5 \end{vmatrix}$$

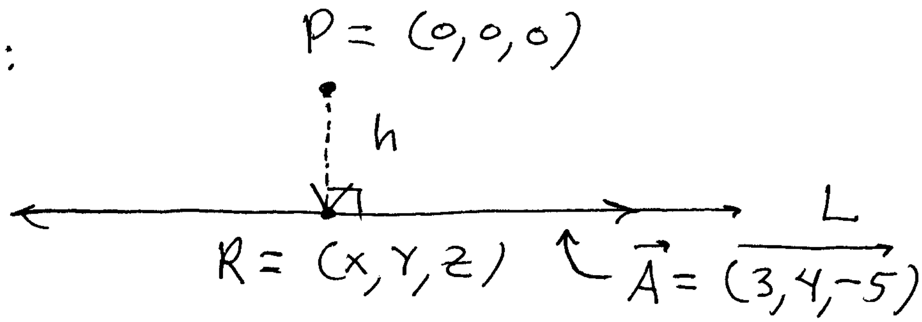
$$= (25-12)\vec{i} - (25-9)\vec{j} + (-20+15)\vec{k}$$

$$= 13\vec{i} - 16\vec{j} - 5\vec{k} \,,$$ so

$$|\vec{QP} \times \vec{A}| = \sqrt{169+256+25} = \sqrt{450} = 15\sqrt{2} \,;$$

then $h = \dfrac{|\vec{QP} \times \vec{A}|}{|\vec{A}|} = \dfrac{15\sqrt{2}}{\sqrt{50}} = \dfrac{15\sqrt{2}}{5\sqrt{2}} = 3$

Method 2:



Find point $R = (x,y,z)$ on line L so that vector $\vec{PR} = \overrightarrow{(x,y,z)}$ is $\perp$ to $\vec{A}$ $\Rightarrow$ $\vec{PR} \cdot \vec{A} = 0 \Rightarrow$

$$\overrightarrow{(x,y,z)} \cdot \overrightarrow{(3,4,-5)} = 0 \Rightarrow$$

$$3x + 4y - 5z = 0 \Rightarrow$$

$$3(5+3t) + 4(5+4t) - 5(-3-5t) = 0 \Rightarrow$$

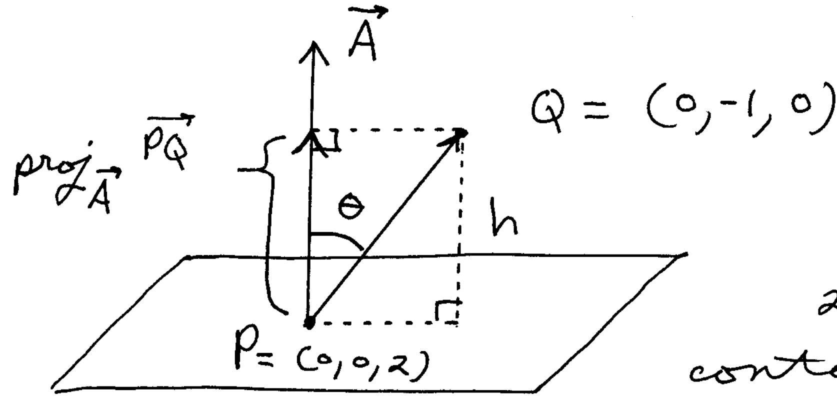
$$15 + 9t + 20 + 16t + 15 + 25t = 0 \Rightarrow$$

$$50t + 50 = 0 \longrightarrow \underline{\underline{t = -1}} \,;$$ then

point $R = (2,1,2)$ and distance from point P to R is

$$h = \sqrt{(2-0)^2 + (1-0)^2 + (2-0)^2} = \sqrt{9} = 3$$

43.)



plane $2X+Y+2Z=4$ contains the point $P=(0,0,2)$; vector $\vec{A} = \overrightarrow{(2,1,2)}$ is $\perp$ to plane ; then distance from Q to plane is

$$h = |proj_{\vec{A}} \vec{PQ}|$$

$$= |\vec{PQ}| \cdot |\cos\theta|$$

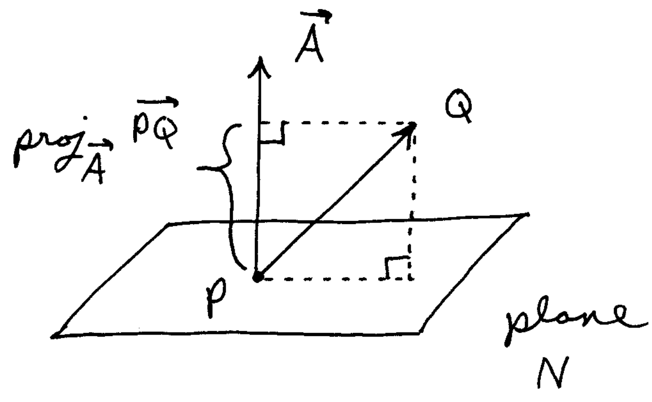
$$= \cancel{|\vec{PQ}|} \cdot \frac{|\vec{PQ} \cdot \vec{A}|}{\cancel{|\vec{PQ}|}\,|\vec{A}|}$$

$$= \frac{|\overrightarrow{(0,-1,-2)} \cdot \overrightarrow{(2,1,2)}|}{|\overrightarrow{(2,1,2)}|}$$

$$= \frac{|0-1-4|}{\sqrt{9}} = 5/3$$

45.) Planes $M: X+2Y+6Z=1$ and $N: X+2Y+6Z=10$ are $\parallel$; point $Q=(1,0,0)$ is on M and point $P=(10,0,0)$ is on N ; vector $\vec{A} = \overrightarrow{(1,2,6)}$ is $\perp$ to N ; now find distance from point Q to plane N ; then distance

$$h = |proj_{\vec{A}} \vec{PQ}|$$



$$= |\vec{PQ}|\,|\cos\theta|$$

$$= \cancel{|\vec{PQ}|} \cdot \frac{|\vec{PQ}\cdot\vec{A}|}{\cancel{|\vec{PQ}|}\,|\vec{A}|}$$

$$= \frac{|\overrightarrow{(-9,0,0)}\cdot\overrightarrow{(1,2,6)}|}{|\overrightarrow{(1,2,6)}|} = \frac{|-9|}{\sqrt{41}} = \frac{9}{\sqrt{41}}$$

47.) Plane $X+Y=1$ has $\perp$ vector $\vec{A} = \overrightarrow{(1,1,0)}$; plane $2X+Y-2Z=2$ has $\perp$ vector $\vec{B} = \overrightarrow{(2,1,-2)}$; the angle between the planes is the angle (acute : $0° \le \theta \le 90°$) determined by the $\perp$ vectors :

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{2+1-0}{\sqrt{2}\cdot\sqrt{9}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$\theta = 45°$

55.) plane $X+Y+Z=2$, line L : $\begin{cases} X = 1+2t \\ y = 1+5t \\ Z = 3t \end{cases}$;

the point of $\cap$ is given by

$X+Y+Z = (1+2t)+(1+5t)+(3t) = 2 \Rightarrow$

$10t = 0 \rightarrow t = 0 \rightarrow$ point is

$(x,y,z) = (1,1,0)$

59.) Plane $x-2y+4z=2$ has $\perp$ vector $\vec{A}=\overrightarrow{(1,-2,4)}$; plane $x+y-2z=5$ has $\perp$ vector $\vec{B}=\overrightarrow{(1,1,-2)}$; then the line forming the $\cap$ of these planes is $\parallel$ to the vector

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 4\\ 1 & 1 & -2\end{vmatrix}=(4-4)\vec{i}-(-2-4)\vec{j}+(1+2)\vec{k}$$

$$=\underline{\underline{6\vec{j}+3\vec{k}}}$$; now find a point on this line :

$$\left.\begin{aligned}x-2y+4z&=2\\ x+y-2z&=5\end{aligned}\right\}\quad \left.\begin{aligned}&\underline{x=2+2y-4z}\\ &x=5-y+2z\end{aligned}\right\}$$

$2+2y-4z=5-y+2z \rightarrow$

$3y=3+6z \longrightarrow \underline{y=1+2z}$; now let z be ANY number : $z=0 \rightarrow$ $y=1 \longrightarrow x=4$, so point $(x,y,z)=\underline{\underline{(4,1,0)}}$ lies on BOTH planes ;

now the line of intersection is given by

$$L:\begin{cases}x=4+(0)t &= 4\\ y=1+(6)t &= 1+6t\\ z=0+(3)t &= 3t\end{cases}$$

67.) Line $L:\begin{cases}x=1-2t\\ y=2+5t\\ z=-3t\end{cases}$ has $\parallel$ vector

$\vec{A} = \overrightarrow{(-2, 5, -3)}$; plane $2X+Y-Z=8$ has $\perp$ vector $\vec{B} = \overrightarrow{(2, 1, -1)}$; then line and plane are parallel if and only if $\vec{A} \perp \vec{B}$:

$\vec{A} \cdot \vec{B} = -4+5+3 = 4 \neq 0$ so

$\vec{A}$ not $\perp$ $\vec{B}$ and line is NOT parallel to plane .