Section 14.2

1.) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2} = \frac{0-0+5}{0+0+2} = \frac{5}{2}$

4.) $\displaystyle \lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2 = \left(\frac{1}{2}+\frac{-1}{3}\right)^2 = \frac{1}{36}$

9.) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{e^y \cdot \sin x}{x} = \lim_{(x,y)\to(0,0)} e^y \cdot \left(\frac{\sin x}{x}\right)$

$= e^0 \cdot (1) = (1)(1) = 1$

12.) $\displaystyle \lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y+1}{y-\sin x} = \frac{\cos 0+1}{0-\sin\frac{\pi}{2}} = \frac{1+1}{-1} = -2$

14.) $\displaystyle \lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = 1+1 = 2$

16.) $\displaystyle \lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y-xy+4x^2-4x}$

$\displaystyle = \lim_{(x,y)\to(2,-4)} \frac{y+4}{xy(x-1)+4x(x-1)}$

$\displaystyle = \lim_{(x,y)\to(2,-4)} \frac{y+4}{(x-1)[xy+4x]}$

$\displaystyle = \lim_{(x,y)\to(2,-4)} \frac{\cancel{y+4}}{x(x-1)[\cancel{y+4}]} = \frac{1}{2(1)} = \frac{1}{2}$

20.) $\displaystyle \lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \cdot \frac{\sqrt{x}+\sqrt{y+1}}{\sqrt{x}+\sqrt{y+1}}$

$$= \lim_{(x,y)\to(4,3)} \frac{x-(y+1)}{(x-y-1)(\sqrt{x}+\sqrt{y+1})} = \lim_{(x,y)\to(4,3)} \frac{\cancel{x-y-1}}{\cancel{(x-y-1)}(\sqrt{x}+\sqrt{y+1})}$$

$$= \frac{1}{2+2} = \frac{1}{4}$$

41) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}}$ DNE since

<u>along path $y=0$</u>:

$$\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2}}$$

$$= \lim_{x\to 0} \frac{-x}{|x|} = \begin{cases} \lim_{x\to 0} \frac{-x}{x} = \lim_{x\to 0} -1 = -1 & \text{if } x>0 \\ \lim_{x\to 0} \frac{-x}{-x} = \lim_{x\to 0} 1 = 1 & \text{if } x<0 \end{cases}$$

so $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}}$ DNE along path $y=0$.

42) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$ DNE since

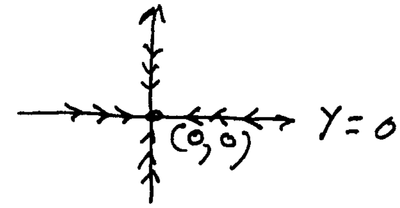
<u>along path $y=0$</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4} = \lim_{x\to 0} 1 = 1;$$

<u>along path $x=0$</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{0+y^2} = \lim_{y\to 0} 0 = 0.$$

43) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ DNE since



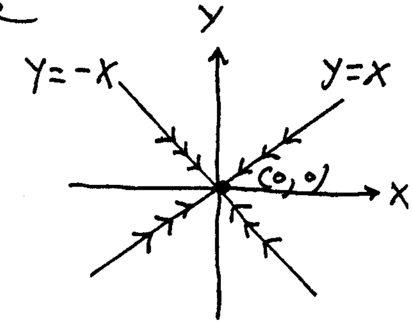
<u>along path Y=0</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4} = \lim_{x\to 0} 1 = 1 \;;$$

<u>along path X=0</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{-y^2}{y^2} = \lim_{y\to 0} -1 = -1$$

44) $\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$ DNE since



<u>along path Y=X</u>:

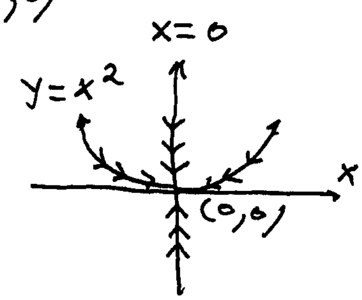
$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,y)\to(0,0)} \frac{x^2}{|x^2|}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2} = \lim_{(x,y)\to(0,0)} 1 = 1 \;;$$

<u>along path Y=-X</u>:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,y)\to(0,0)} \frac{-x^2}{x^2} = \lim_{(x,y)\to(0,0)} -1 = -1 .$$

47) $\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y}$ DNE since



<u>along path X=0</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(x,y)\to(0,0)} \frac{y}{y} = \lim_{y\to 0} 1 = 1 \;;$$

<u>along path Y=X²</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(x,y)\to(0,0)} \frac{2x^2}{x^2} = \lim_{x\to 0} 2 = 2 .$$

Chapter 14
Practice Exercises

12.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^3y^3-1}{xy-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(1,1)} \frac{(xy)^3-(1)^3}{xy-1}$

$\displaystyle = \lim_{(x,y)\to(1,1)} \frac{\cancel{(xy-1)}\left((xy)^2+(xy)+1\right)}{\cancel{xy-1}}$

$= 1 + 1 + 1 = 3$

16.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy}$ DNE since

<u>along path $y=x$</u> :

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy} = \lim_{(x,y)\to(0,0)} \frac{2x^2}{x^2} = \lim_{x\to0} 2 = 2$;

<u>along path $y=-x$</u> :

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy} = \lim_{(x,y)\to(0,0)} \frac{2x^2}{-x^2} = \lim_{x\to0} -2 = -2$