Section 10.2

7) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} = 1 + \frac{-1}{4} + \frac{1}{4^2} + \frac{-1}{4^3} + \frac{1}{4^4} + \frac{-1}{4^5} + \cdots$$
$$= 1 + \left(\frac{-1}{4}\right) + \left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{4}\right)^3 + \left(\frac{-1}{4}\right)^4 + \cdots$$
$$= \frac{1}{1-\left(\frac{-1}{4}\right)} = \frac{1}{5/4} = \frac{4}{5}$$

8) $$\sum_{n=2}^{\infty} \frac{1}{4^n} = \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5} + \cdots$$
$$= \frac{1}{4^2} \cdot \left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots\right]$$
$$= \frac{1}{16} \cdot \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{1}{16} \cdot \frac{4}{3} = \frac{1}{12}$$

12) $$\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n}\right) = 5\sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n}$$
$$= 5\left(1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \cdots\right) - \left(1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \cdots\right)$$
$$= 5 \cdot \frac{1}{1-\left(\frac{1}{2}\right)} - \frac{1}{1-\left(\frac{1}{3}\right)}$$
$$= 5 \cdot (2) - \frac{3}{2} = \frac{17}{2}$$

14) $$\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right) = \sum_{n=0}^{\infty} 2 \cdot \frac{2^n}{5^n}$$
$$= 2 \cdot \left[1 + \left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots\right]$$
$$= 2 \cdot \frac{1}{1-\frac{2}{5}} = 2 \cdot \frac{5}{3} = \frac{10}{3}$$

42) $$\frac{6}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$$= \frac{A(2n+1)+B(2n-1)}{(2n-1)(2n+1)} \longrightarrow$$

$A(2n+1)+B(2n-1)=6$;

Let $x=\frac{1}{2}$: $2A=6 \rightarrow A=3$,

Let $x=\frac{-1}{2}$: $-2B=6 \rightarrow B=-3$; ~~then~~

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty}\left(\frac{3}{2n-1}+\frac{-3}{2n+1}\right) ;$$

$S_1 = 3+(-1)=2$,

$S_2 = (3+\cancel{(-1)})+\cancel{(1}+(-3/5)) = 3-3/5$,

$S_3 = (3+\cancel{(-1)})+\cancel{(1+(\frac{-3}{5})})+\cancel{(\frac{3}{5}}+(\frac{-3}{7})) = 3-\frac{3}{7}$,

$S_4 = (3+\cancel{(-1)})+\cancel{(1+(\frac{-3}{5})})+\cancel{(\frac{3}{5}+(\frac{-3}{7})})+\cancel{(\frac{3}{7}}+(\frac{-3}{9})) = 3-\frac{3}{9}$,

$\vdots$

$S_n = 3-\frac{3}{2n+1}$; and by sequence of partial sums series converges since

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \lim_{n\to\infty}\left(3-\frac{3}{2n+1}\right) = 3-0 = 3$$

45) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$;

$S_1 = 1-\frac{1}{\sqrt{2}}$,

$S_2 = (1-\cancel{\frac{1}{\sqrt{2}}})+(\cancel{\frac{1}{\sqrt{2}}}-\frac{1}{\sqrt{3}}) = 1-\frac{1}{\sqrt{3}}$,

$S_3 = (1-\cancel{\frac{1}{\sqrt{2}}})+(\cancel{\frac{1}{\sqrt{2}}}-\cancel{\frac{1}{\sqrt{3}}})+(\cancel{\frac{1}{\sqrt{3}}}-\frac{1}{\sqrt{4}}) = 1-\frac{1}{\sqrt{4}}$,

$\vdots$

$s_n = 1 - \frac{1}{\sqrt{n+1}}$; and by sequence of partial sums series <u>converges</u> since

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = \lim_{n\to\infty}\left(1 - \frac{1}{\sqrt{n+1}}\right)$$

$$= 1 - 0 = 1 .$$

50) $\sum_{n=0}^{\infty} (\sqrt{2})^n$; $\lim_{n\to\infty} (\sqrt{2})^n = \infty \neq 0$

so series <u>diverges</u> by nth-term test.

51) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot (-1) \cdot \frac{3}{2^n}$

$= -3 \cdot \sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n$ so series <u>converges</u>

by geometric series test since $r = -\frac{1}{2}$ and $-1 < r < 1$ with value

$$-3 \sum_{n=1}^{\infty}\left(\frac{-1}{2}\right)^n = -3 \cdot \left[\left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \cdots\right]$$

$$= (-3)\left(\frac{-1}{2}\right) \cdot \left[1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \cdots\right]$$

$$= \frac{3}{2} \cdot \frac{1}{1 - \left(\frac{-1}{2}\right)} = \frac{3}{2} \cdot \frac{2}{3} = 1$$

52) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot n$; $\lim_{n\to\infty} (-1)^{n+1} \cdot n \neq 0$

so series <u>diverges</u> by nth-term test.

54) $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n} = \frac{\cos 0}{1} + \frac{\cos \pi}{5} + \frac{\cos 2\pi}{5^2} + \cdots$

$= 1 - \frac{1}{5} + \frac{1}{5^2} - \frac{1}{5^3} + \cdots$

$= 1 + \left(\frac{-1}{5}\right) + \left(\frac{-1}{5}\right)^2 + \left(\frac{-1}{5}\right)^3 + \cdots$

$= \frac{1}{1-(-1/5)} = \frac{5}{6}$ so series converges by geometric series test since $r = -1/5$ and $-1 < r < 1$.

60) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$; $\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n$

$= \lim_{n\to\infty} \left(1 + \frac{(-1)}{n}\right)^n = e^{-1} \neq 0$ so series diverges by nth-term test

62) $\sum_{n=1}^{\infty} \frac{n^n}{n!} = \frac{1}{1} + \frac{2\cdot 2}{2\cdot 1} + \frac{3\cdot 3\cdot 3}{3\cdot 2\cdot 1} + \frac{4\cdot 4\cdot 4\cdot 4}{4\cdot 3\cdot 2\cdot 1} + \cdots$

$> 1 + 1 + 1 + 1 + \cdots$ so series diverges by comparison to a divergent series

63) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} (\ln n - \ln(n+1))$;

$S_1 = \cancel{\ln 1} - \ln 2 = -\ln 2$,

$S_2 = (\cancel{\ln 1} - \cancel{\ln 2}) + (\cancel{\ln 2} - \ln 3) = -\ln 3$,

$S_3 = (\cancel{\ln 1} - \cancel{\ln 2}) + (\cancel{\ln 2} - \cancel{\ln 3}) + (\cancel{\ln 3} - \ln 4)$

$= -\ln 4$,

$\vdots$

$S_n = -\ln(n+1)$; and by sequence of partial sums series <u>diverges</u> since

$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} -\ln(n+1) = -\infty$

67) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n = 1 + \left(\frac{e}{\pi}\right) + \left(\frac{e}{\pi}\right)^2 + \left(\frac{e}{\pi}\right)^3 + \cdots$

$= \frac{1}{1-\left(\frac{e}{\pi}\right)} = \frac{\pi}{\pi - e}$ so series

<u>converges</u> by geometric series test since $r = \frac{e}{\pi}$ and $-1 < r < 1$.

20) $0.234234234\ldots$

$= \frac{234}{1000} + \frac{234}{1{,}000{,}000} + \frac{234}{1{,}000{,}000{,}000} + \cdots$

$= 234\left(\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}} + \cdots\right)$

$= (234)\left(\frac{1}{10^3}\right) \cdot \left[1 + \frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \cdots\right]$

$$= \frac{234}{1000} \cdot \left[1 + \left(\frac{1}{1000}\right) + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{1000}\right)^3 + \cdots \right]$$

$$= \frac{234}{1000} \cdot \frac{1}{1 - \frac{1}{1000}} = \frac{234}{999} = \frac{26}{111}$$

21) $0.7777\cdots$

$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \frac{7}{10,000} + \cdots$$

$$= \frac{7}{10} \cdot \left[1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \cdots \right]$$

$$= \frac{7}{10} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{7}{10} \cdot \frac{10}{9} = \frac{7}{9}$$

86) If Σa_n converges and $a_n > 0$, then $\lim_{n\to\infty} a_n = 0$. Thus, $\lim_{n\to\infty} \frac{1}{a_n} = \text{"}\frac{1}{0^+}\text{"} = +\infty \neq 0$ so $\Sigma \frac{1}{a_n}$ diverges by the nth-term test.

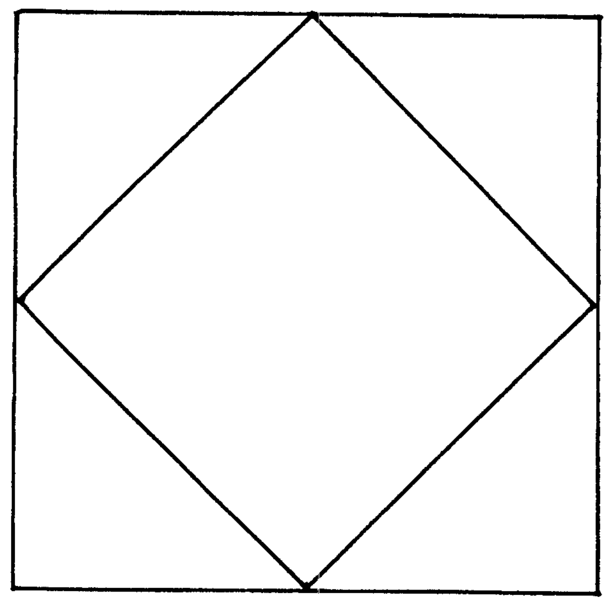
90) $1 + e^b + e^{2b} + e^{3b} + \cdots = 9 \rightarrow$

$1 + (e^b) + (e^b)^2 + (e^b)^3 + \cdots = 9 \rightarrow$

$\frac{1}{1 - e^b} = 9 \rightarrow \frac{1}{9} = 1 - e^b \rightarrow$

$e^b = \frac{8}{9} \rightarrow b = \ln(8/9)$.

93)



assume big square is $\underline{\underline{x \text{ by } x}}$, then area of inscribed square is

$$\text{Area} = x^2 - 4\left(\frac{x}{2}\right)^2 \cdot \frac{1}{2} = \frac{1}{2}x^2 ;$$

inscribed square is $\underline{\underline{\frac{1}{\sqrt{2}}x \text{ by } \frac{1}{\sqrt{2}}x}}$;

then sum of areas of all squares is

$$S = (2)^2 + \left(\frac{1}{\sqrt{2}}\cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^2 \cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^3 \cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^4 \cdot 2\right)^2 + \cdots$$

$$= 4 + \left(\frac{1}{\sqrt{2}}\right)^2 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^4 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^6 \cdot (4) + \cdots$$

$$= 4 \cdot \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right]$$

$$= 4 \cdot \left[1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots\right]$$

$$= 4 \cdot \frac{1}{1 - \left(\frac{1}{2}\right)} = 8 \text{ m.}^2$$