Section 14.6

1.) $\underbrace{x^2+y^2+z^2}_{f(x,y,z)}=3$ and $P=(1,1,1)$;

find gradient vector :

$f_x=2x,\ f_y=2y,\ f_z=2z$ so

$\vec{\nabla}f(1,1,1)=f_x(1,1,1)\vec{i}+f_y(1,1,1)\vec{j}+f_z(1,1,1)\vec{k} \Rightarrow$

$\vec{\nabla}f(1,1,1)=2\vec{i}+2\vec{j}+2\vec{k}$; then

a.) tangent plane :

$2(x-1)+2(y-1)+2(z-1)=0 \Rightarrow x+y+z=3$.

b.) normal line :

$$L:\begin{cases} x=1+2t \\ y=1+2t \\ z=1+2t \end{cases}$$

4.) $\underbrace{x^2+2xy-y^2+z^2}_{f(x,y,z)}=7$ and $P=(1,-1,3)$;

find gradient vector :

$f_x=2x+2y,\ f_y=2x-2y,\ f_z=2z$ so

$\vec{\nabla}f(1,-1,3)=f_x(1,-1,3)\vec{i}+f_y(1,-1,3)\vec{j}+f_z(1,-1,3)\vec{k} \Rightarrow$

$\vec{\nabla}f(1,-1,3)=(0)\vec{i}+(4)\vec{j}+(6)\vec{k}=4\vec{j}+6\vec{k}$;

then

a.) tangent plane :

$(0)(x-1)+4(y+1)+6(z-3)=0 \Rightarrow$

$4y+4+6z-18=0 \Rightarrow 4y+6z=14 \Rightarrow$

$2y+3z=7$.

b.) normal line :

$$L: \begin{cases} x = 1 + (0)t \\ y = -1 + (4)t \\ z = 3 + (6)t \end{cases} \Rightarrow L: \begin{cases} x = 1 \\ y = -1 + 4t \\ z = 3 + 6t \end{cases}$$

9.) $z = \ln(x^2+y^2) \Rightarrow \underbrace{z - \ln(x^2+y^2)}_{f(x,y,z)} = 0$

and $P = (1,0,0)$;

find gradient vector :

$f_x = \dfrac{-2x}{x^2+y^2}$, $f_y = \dfrac{-2y}{x^2+y^2}$, $f_z = 1$ so

$\vec{\nabla} f(1,0,0) = f_x(1,0,0)\,\vec{i} + f_y(1,0,0)\,\vec{j} + f_z(1,0,0)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,0,0) = (-2)\vec{i} + (0)\,\vec{j} + (1)\,\vec{k} = -2\vec{i} + \vec{k}$;

then tangent plane is

$(-2)(x-1) + (0)(y-0) + (1)(z-0) = 0 \Rightarrow$

$-2x + 2 + z = 0 \Rightarrow z = 2x - 2$

12.) $z = 4x^2 + y^2 \Rightarrow \underbrace{z - 4x^2 - y^2}_{f(x,y,z)} = 0$

and $P = (1,1,5)$;

find gradient vector :

$f_x = -8x$, $f_y = -2y$, $f_z = 1$ so

$\vec{\nabla} f(1,1,5) = f_x(1,1,5)\,\vec{i} + f_y(1,1,5)\,\vec{j} + f_z(1,1,5)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,1,5) = -8\vec{i} - 2\vec{j} + \vec{k}$; then tangent plane

is $-8(x-1) - 2(y-1) + (z-5) = 0 \Rightarrow$

$-8x + 8 - 2y + 2 + z - 5 = 0 \Rightarrow -8x - 2y + z = -5$

14.) $\underbrace{xyz}_{f(x,y,z)} = 1$ and $\underbrace{x^2+2y^2+3z^2}_{g(x,y,z)} = 6$

and $P = (1,1,1)$; find gradient vectors:

$f_x = yz$, $f_y = xz$, $f_z = xy$ $\Rightarrow$

$\vec{\nabla} f(1,1,1) = (1)\vec{i} + (1)\vec{j} + (1)\vec{k} = \underline{\underline{\vec{i}+\vec{j}+\vec{k}}}$;

$g_x = 2x$, $g_y = 4y$, $g_z = 6z$ $\Rightarrow$

$\vec{\nabla} g(1,1,1) = (2)\vec{i} + (4)\vec{j} + (6)\vec{k} = \underline{\underline{2\vec{i}+4\vec{j}+6\vec{k}}}$;

vector $\vec{\nabla} f(1,1,1)$ is $\perp$ to level surface $f(x,y,z) = 1$ at $(1,1,1)$, and vector $\vec{\nabla} g(1,1,1)$ is $\perp$ to level surface $g(x,y,z) = 6$, so vector

$$\boxed{\vec{T} = \vec{\nabla} f(1,1,1) \times \vec{\nabla} g(1,1,1)}$$

is tangent to both curves (curve of intersection) at $\underline{\underline{P = (1,1,1)}}$; then

$$\vec{T} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = (6-4)\vec{i} - (6-2)\vec{j} + (4-2)\vec{k}$$
$$= 2\vec{i} - 4\vec{j} + 2\vec{k} ,$$

so tangent line is

$$L: \begin{cases} x = 1 + (2)t \\ y = 1 + (-4)t \\ z = 1 + (2)t \end{cases} \Rightarrow L: \begin{cases} x = 1+2t \\ y = 1-4t \\ z = 1+2t \end{cases}$$

15.) $\underbrace{x^2+2y+2z}_{f(x,y,z)} = 4$ and $\underbrace{y}_{g(x,y,z)} = 1$

and $P = (1,1,\tfrac{1}{2})$; find gradient vectors :

$f_x = 2x$, $f_y = 2$, $f_z = 2$ so

$\vec{\nabla} f(1,1,\tfrac{1}{2}) = 2\vec{i} + 2\vec{j} + 2\vec{k}$;

$g_x = 0$, $g_y = 1$, $g_z = 0$ so

$\vec{\nabla} g(1,1,\tfrac{1}{2}) = (0)\vec{i} + (1)\vec{j} + (0)\vec{k} = \vec{j}$; then

$$\vec{T} = \vec{\nabla} f(1,1,\tfrac{1}{2}) \times \vec{\nabla} g(1,1,\tfrac{1}{2}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2 \\ 0 & 1 & 0 \end{vmatrix}$$

$= (0-2)\vec{i} - (0-0)\vec{j} + (2-0)\vec{k} = -2\vec{i} + 2\vec{k}$;

so tangent line is

$$L: \begin{cases} x = 1 + (-2)t \\ y = 1 + (0)t \\ z = \frac{1}{2} + (2)t \end{cases} \Rightarrow L: \begin{cases} x = 1 - 2t \\ y = 1 \\ z = \frac{1}{2} + 2t . \end{cases}$$

20.) $f(x,y,z) = e^x \cos yz$, $P = (0,0,0)$,

$ds = 0.1$, and $\vec{A} = 2\vec{i} + 2\vec{j} - 2\vec{k}$; then

$\vec{u} = \frac{1}{|\vec{A}|}\vec{A} = \frac{1}{2\sqrt{3}}(2\vec{i} + 2\vec{j} - 2\vec{k}) \Rightarrow$

$\vec{u} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}$; find

gradient vector :

$f_x = e^x \cos yz$, $f_y = -ze^x \sin yz$, $f_z = -ye^x \sin yz$

so $\vec{\nabla} f(0,0,0) = (1)\vec{i} + (0)\vec{j} + (0)\vec{k} = \vec{i}$; then the <u>differential</u> is

$$df = \left(D_{\vec{u}} f(0,0,0)\right) \cdot ds$$
$$= \left(\vec{\nabla} f(0,0,0) \bullet \vec{u}\right) \cdot ds$$
$$= \left(\vec{i} \bullet \left(\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)\right) \cdot (0.1)$$
$$= \frac{1}{\sqrt{3}}(0.1) \approx 0.0577$$

21.) $g(x,y,z) = x + x\cos z - y\sin z + y$, $P = (2,-1,0)$, $ds = 0.2$, move toward $Q = (0,1,2)$ so $\overrightarrow{PQ} = \overrightarrow{(-2,2,2)}$; then $\vec{u} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} \Rightarrow$

$\vec{u} = \frac{1}{2\sqrt{3}}\overrightarrow{(-2,2,2)} = \overrightarrow{\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$;

find gradient vector :

$g_x = 1 + \cos z$, $g_y = -\sin z + 1$,
$g_z = -x\sin z - y\cos z$ so

$\vec{\nabla} g(2,-1,0) = (2)\vec{i} + (1)\vec{j} + (1)\vec{k} = 2\vec{i} + \vec{j} + \vec{k}$; then the <u>differential</u> is

$$df = \left(D_{\vec{u}}\, g(2,-1,0)\right) \cdot ds$$
$$= \left(\vec{\nabla} g(2,-1,0) \bullet \vec{u}\right) \cdot ds$$

$$= \left((2\vec{i} + \vec{j} + \vec{k}) \cdot \left(-\frac{1}{\sqrt{3}} \vec{i} + \frac{1}{\sqrt{3}} \vec{j} + \frac{1}{\sqrt{3}} \vec{k} \right) \right) \cdot (0.2)$$

$$= \left(\frac{-2}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right) (0.2) = (0)(0.2) = 0 .$$