Math 022B Midterm 2 Nov. 21, 2008

Name:______________________ ID ________________________

No Calculators allowed. Use the back of all the pages if needed.

(1) (16 points) (a) Find the solution $y(t)$ of the initial value problem

$$4y'' - y = 0, \quad y(0) = \alpha,\ y'(0) = 2.$$

(b)Find α so that the solution approaches zero as $t \to \infty$.

(2) (22 points) (a) Verify that $y_1(t) = t^2$ is a solution of the ODE

$$t^2 y'' - 2y = 0\,(t > 0).$$

(b) Use the method of reduction of order to find a second solution $y_2(t)$ and verify that $\{y_1, y_2\}$ form a fundamental set of solutions of the ODE.

(3) (26 points) (a) Find a particular solution of the ODE by the method of undetermined coefficients:

$$y'' + 2y' + y = 3te^{-t}$$

(b)Find the general solution.

(c)Re-do part (a) by the method of variation of parameters.

(4) (16 points) Use the Laplace transform to solve the initial value problem:

$$y'' + 2y' + y = 4e^{-t}, \quad y(0) = 2,\ y'(0) = -1.$$

(Hint: you can break your function into several parts and use $\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$ to do the inverse Laplace transform)

(5) (20 points) (a) Find the general solution of the following system of equations and sketch the phase portrait of the system.

$$\mathbf{X}' = \begin{pmatrix} 1/2 & 3/4 \\ 3 & 1/2 \end{pmatrix} \mathbf{X}$$

(b) Find the solution that satisfies the initial condition:

$$\mathbf{X}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$