Mat127a Discussion Section 6

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Exercise 14.1 See page 321.

Exercise 14.2 (a) Compare with $\sum \frac{n}{2n^2}$.
(b) Use Corollary 14.5.
(c) Use Theorem 15.1.
(d), (e), (g) Use Ratio test.
(f) Use Root test.

Exercise 14.5 See page 321.

Exercise 14.7 See page 322.

Exercise 14.10 It suffices to find a sequence (a_n) of nonzero numbers with the following property:

$$\liminf \left|\frac{a_{n+1}}{a_n}\right| \le 1 < \limsup |a_n|^{\frac{1}{n}} \le \limsup \left|\frac{a_{n+1}}{a_n}\right|.$$

Verify that the sequence $a_n = 2^{(-1)^n+n}$ satisfies this property. (*cf*. Example 8)

Exercise 14.13 (a), (b) See page 322. (c) Note that

$$\sum_{k=1}^{n} \frac{k-1}{2^{k+1}} = \sum_{k=1}^{n} \left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right) = \llcorner\_\_\_\_\_\_\_\_\_\lrcorner.$$

Since $\lim \frac{n}{2^n} = 0$ (Exercise 9.12), we have

$$\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{k-1}{2^{k+1}} = \lim_{n\to\infty} (\llcorner\_\_\_\_\_\_\_\_\_\lrcorner) = \frac{1}{2}.$$

(d)

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 4\sum_{n=1}^{\infty} \frac{n}{2^{n+2}} = 4\sum_{n=2}^{\infty} \frac{n-1}{2^{n+1}} = 4\left(\frac{1}{2} - 0\right) = 2.$$

Exercise 15.2 (a) Use Corollary 14.5.

(b) Use Root test. What can you say about the value of $\limsup \left|\left(\sin \frac{n\pi}{7}\right)^n\right|^{\frac{1}{n}}$?

Exercise 15.6 (a) $a_n = \frac{1}{n}$; (c) $a_n = \frac{(-1)^n}{\sqrt{n}}$.

Just for fun.

Exercise 14.6 (a) If $|b_n| \le M$, then

$$\left|\sum_{k=m}^{n} a_k b_k\right| \le \sum_{k=m}^{n} |a_k||b_k| \le M \sum_{k=m}^{n} |a_k|.$$

Exercise 14.14 Observe that

$$\begin{aligned}
&\frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \cdots\right. \\
&> \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{16} + \cdots\right. \\
&= \frac{1}{2} + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) + \cdots
\end{aligned}$$

Exercise 15.3 Let $u = \log x$. Then $\frac{du}{dx} = \frac{1}{x}$ and thus

$$\int \frac{1}{x(\log x)^p} dx = \int \frac{1}{u^p} du = \frac{1}{1-p} \frac{1}{u^{p-1}} + C.$$

Exercise 15.7 See page 322.