Mat127a Homework Solutions 3

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Exercise 10.1 (a) nonincreasing and bounded; (b) bounded; (c) nondecreasing; (d) bounded; (e) none of the three; (f) nonincreasing and bounded.

Exercise 10.4 As is explained in the following paragraphs of Theorem 10.2 and 10.11, these two theorems rely on the Completeness Axiom 4.4 for the set $\mathbb{R}$ of real numbers, which does not hold for the set $\mathbb{Q}$ of rational numbers.

For an explicit counterexample, consider the sequence $c_n = (1 + \frac{1}{n})^n$ in Example 1. It is a sequence of rational numbers which is (i) bounded and nondecreasing and also (ii) Cauchy, but does not converge to a rational number.

Exercise 10.5 Let (s_n) be an unbounded nonincreasing sequence. Let $M < 0$. Since the set $\{s_n \mid n \in \mathbb{N}\}$ is unbounded and it is bounded above by s_1, it must be unbounded below. Hence for some N in $\mathbb{N}$ we have $s_N < M$. Clearly $n > N$ implies $s_n \leq s_N < M$, so $\lim s_n = -\infty$. (*cf.* Definition 9.8)

Exercise 10.7 See page 317.