Homework 2

due January 23, 2015 for presentation in class

1. An inversion in $\pi = \pi_1\pi_2\cdots\pi_n \in S_n$ in one-line notation is a pair π_i, π_j such that $i < j$ and $\pi_i > \pi_j$. Let $\mathrm{inv}(\pi)$ be the number of inversions of π.

(a) Show that if π can be written as a product of k transpositions, then $k \equiv \mathrm{inv}(\pi) \pmod 2$.

(b) Use part (a) to show that the sign of π is well-defined.

2. Let G act on S with corresponding permutation representation $\mathbb{C}S$. Prove the following:

(a) The matrices for the action of G in the standard basis (meaning, with the elements of S as the basis) are permutation matrices.

(b) If the character of this representation is χ and $g \in G$, then $\chi(g)$ is the number of fixed points of g acting on S.

3. Sage exercise:
Write a Sage program where you input n (any positive integer) and a partition λ of n, and the program returns the value of the character $\chi^{\mathrm{def}}(\lambda)$ for the defining representation.