Homework 4

due February 6, 2015 for presentation in class

1. Let χ^λ be the character of M^λ. Find (with proof) a formula for $\chi^\lambda(\lambda)$, the value of χ^λ on the conjugacy class K_λ.

2. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_\ell)$ and $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ be partitions. Characterize the fact that λ is covered by μ if the ordering used is (a) lexicographic, (b) dominance order.

3. Consider $S^{(n-1,1)}$, where each tabloid is identified with the element in its second row. Prove the following facts about this module and its character.

(a) We have

$$S^{(n-1,1)} = \{c_1\mathbf{1} + c_2\mathbf{2} + \cdots + c_n\mathbf{n} \mid c_1 + \cdots + c_n = 0\}.$$

(b) For any $\pi \in S_n$,

$$\chi^{(n-1,1)}(\pi) = \text{number of fixed points of } \pi - 1.$$