Homework 7

due March 6, 2015 for presentation in class

1. The symmetric group S_n is generated by the simple transpositions $s_1, s_2, \ldots, s_{n-1}$, where $s_i = (i, i+1)$. The Jucys–Murphy elements are elements in the group algebra defined as a sum of transpositions

$$X_i = (1, i) + (2, i) + \cdots + (i-1, i) \qquad \text{for } i = 1, 2, \ldots, n.$$

Show that:

(a)
$$s_i X_i + 1 = X_{i+1} s_i \qquad \text{for } i = 1, 2, \ldots, n-1.$$

(b) The Jucys–Murphy elements commute, that is, $X_i X_j = X_j X_i$ for all $1 \leq i, j < n$.

2. Let s_i for $1 \leq i < n$ be the simple transpositions in S_n and let $u, v \in \mathbb{C}$. Show that in $\mathbb{C}S_n$ the Yang-Baxter equation holds:

$$\big(s_i + \frac{1}{u}\big)\big(s_{i+1} + \frac{1}{u+v}\big)\big(s_i + \frac{1}{v}\big) = \big(s_{i+1} + \frac{1}{v}\big)\big(s_i + \frac{1}{u+v}\big)\big(s_{i+1} + \frac{1}{u}\big).$$