Lecture 16

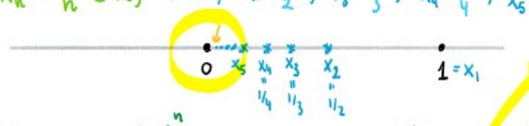
Tuesday, September 29, 2020 12:05 PM

Lecture 16 : Sequences & Limits

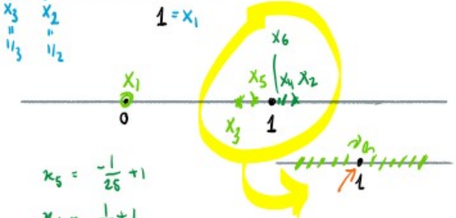
Let $(x_n)_{n\in\mathbb{N}}$ a sequence of real numbers.

Examples: (1) $x_1 = 0.13$, $x_2 = -7.22$, $x_3 = 1.327$, $x_4 = \pi^2$, $x_5 = \log 2$, ...

(2) $x_n = \frac{1}{n} \in \mathbb{R}$, $x_1 = 1$, $x_2 = \frac{1}{2}$, $x_3 = \frac{1}{3}$, $x_4 = \frac{1}{4}$, $x_5 = \frac{1}{5}$, e.g. $x_{1000} = \frac{1}{1000} = 0.001$



(3) $(x_n)_{n\in\mathbb{N}}$, $x_n = \frac{(-1)^n}{n^2} + 1$,



$x_1 = -\frac{1}{1^2} + 1 = 0$ $\quad x_3 = -\frac{1}{9} + 1$ $\quad x_5 = -\frac{1}{25} + 1$

$x_2 = \frac{1}{4} + 1 = 5/4$ $\quad x_4 = \frac{1}{16} + 1$ $\quad x_6 = \frac{1}{36} + 1$

§1. Limits & Convergence

thought of as being a small distance away from L

limit!

Def^n: A sequence of real numbers (x_n) converges to L if
$\exists L \in \mathbb{R}$ such that $\forall \varepsilon > 0$ $\exists n_0 \in \mathbb{N}$ such that
$$|x_n - L| < \varepsilon, \quad \forall n \geq n_0$$ ▨

i.e. x_n lies in ε-vicinity of L: $(L-\varepsilon, L+\varepsilon)$

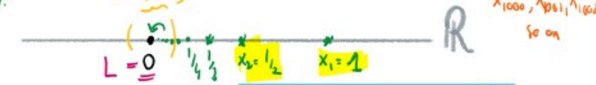
distance between x_n & L.

eventually in the sequence

"true for x_n, if n large enough"

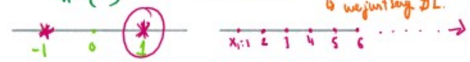
x: $|x - L| < \varepsilon$ = $(L-\varepsilon, L+\varepsilon)$

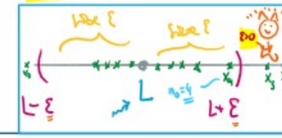
Ex: (i) $x_n = \frac{1}{n}$ converges to 0.



(ii) A sequence (x_n) MIGHT NOT converge:

$x_n = (-1)^n$ no limits $x_n = n$ "limit is ∞" we just say $\nexists L$.





Instance of a limit from definition

Prob: Show that $x_n = 1/n$ converges to 0, i.e. $\lim_{n\to\infty} \frac{1}{n} = 0$.

Sol^n: For any $\varepsilon > 0$, we need to show that $\exists n_0 \in \mathbb{N}$ s.t. $|x_n - 0| < \varepsilon$, $\forall n \geq n_0$. $\Big\}$ here $L = 0$, and we want $\left|\frac{1}{n} - 0\right| < \varepsilon$, $\forall n \geq n_0$.

"n large enough"

This is showing that $\left|\frac{1}{n}\right| < \varepsilon$ for n "large enough",

$\frac{1}{n+1} < \frac{1}{n} < \varepsilon$

use Prop. 10. which says that $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. $\boxed{\frac{1}{N} < \varepsilon}$.

Then we choose $\boxed{n_0 := N}$ because $\frac{1}{n} < \varepsilon$ for any $n \geq N$ □

define n_0 to be N