Lecture 3

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Lecture 3: Principle of Mathematical Induction

we introduced $\mathbb{N} \subseteq \mathbb{Z}$ ← integer, with them we can introduce an order as follows:

natural, a subset

$n > m$ ↕ equiv. $n - m$

Def: Let $n, m \in \mathbb{Z}$. We write $m < n$ if $n + (-m) \in \mathbb{N}$. ⊛

defined by

Corollary: $\mathbb{N} \overset{②}{\underset{\subseteq}{=}} \{n \in \mathbb{Z} \text{ s.t. } n > 0\}$. (equiv. $n - 0 \in \mathbb{N}$) notation

⊛ Proving = between sets means $\subseteq$ and $\supseteq$.

Work through properties of the order "<" defined above, in particular prove Prop. 2.4 through 2.13.

§1. Induction: a method of proof

← useful: PSet 2 has tons of problems solvable by induction

we add one last axiom for $\mathbb{N} \subseteq \mathbb{Z}$:

Axiom 2.15: if a subset $A \subseteq \mathbb{Z}$ satisfies

(i) $1 \in A$ (ii) if $n \in A$ then $n+1 \in A$ } assumption

Then $\mathbb{N} \subseteq A$. ← conclusion

+1

equiv. $\mathbb{N} \subseteq \mathbb{Z}$ is the smallest subset satisfying (i) & (ii)

Ex.: $A = \mathbb{N} \cup \{0\}$ has (i) & (ii)

$A = \mathbb{N} \cup \{-1, 0\}$ has (i) & (ii).

proof outlined

The main application of Axiom 2.15 is:

Theorem 2.17. (Principle of induction) Let $P(n)$ be a statement that depends on $n \in \mathbb{N}$. If

first piece → (1) $P(1)$ is true ← base case

(2) $\forall n \in \mathbb{N}$, $P(n)$ true implies $P(n+1)$ true ← induction step

if one piece falls next falls

Then $P(n)$ is true $\forall n \in \mathbb{N}$.

↳ informally

1 2 3 4 5 6 ...

if this falls

need this one to fall

if true ⇒ true

Example of a problem: $P(n) =$ "7 divides $5^{2n+1} + 2^{2n+1}$ for all $n \in \mathbb{N}$."

(also 2.18 in book & PSet 2) statement depending on $n \in \mathbb{N}$.

$133 = 7 \cdot 19$

$n=1$: $P(1) =$ "7 divides $5^3 + 2^3$." 125 8

$n=2$: $P(2) =$ "7 divides $5^5 + 2^5$." 3157

$n=3$: $P(3) =$ "7 divides $5^7 + 2^7$" and so on.

Two Steps for induction are:

(i) Base case ($n=1$) (ii) Induction Step.

Proof: By induction. The base case is $P(1) =$ "7 divides $5^3 + 2^3$", since $5^3 + 2^3 = 133 = 7 \cdot 19$, the statement $P(1)$ is true. For the induction step, we assume $P(n)$ is true and try to deduce $P(n+1)$ is true. We assume "7 divides $5^{2n+1} + 2^{2n+1}$", we want to show that "7 divides $5^{2n+3} + 2^{2n+3}$" $=: P(n+1)$ is true.