Lecture 9

Tuesday, September 29, 2020 12:05 PM

Lecture 9: Notions of Set Theory ← Chapter 5

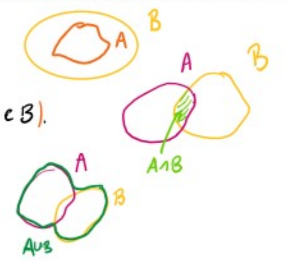
Let X be a set, typically for us $X \subseteq \mathbb{Z} = \{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

Examples: $X_1 := \{a \in \mathbb{Z} : a \text{ is even}\}$ | $X_1 = \{\dots, -4, -2, 0, 2, 4, 6, \dots\}$ | $X_2 \subseteq X_1$ true

$X_2 := \{a \in \mathbb{Z} : a = 4k+2 \text{ for some } k \in \mathbb{Z}\}$ | $X_2 = \{\dots, -2, 2, 6, 10, 14, \dots\}$ | $X_3 \nsubseteq X_1$

$X_3 := \{a \in \mathbb{Z} : a = 6k+5 \text{ for } k \in \mathbb{Z}\}$ | $X_3 = \{\dots, -1, 5, 11, 17, \dots\}$ | $X_2 \cap X_3 = \phi$ or empty set

$X_4 := \{a \in \mathbb{Z} : a = 6k-1 \text{ for } k \in \mathbb{Z}\}$ | $X_4 = \{\dots, -1, 5, 11, 17, \dots\}$ | $X_3 = X_4$ or $X_3 \subseteq X_4$ & $X_4 \subseteq X_3$



Def^n: Let A, B be sets. (i) $A \subseteq B$ iff $\forall x \in A$ satisfies $x \in B$.

(ii) $A \cap B$ is the subset of A, and B, s.t. $x \in A \cap B$ iff $(x \in A$ and $x \in B)$.

(iii) $A \cup B$ is the set defined by $x \in A \cup B$ iff $(x \in A$ or $x \in B)$.

or / and inclusive

Def / Prop.: Two sets A, B are equal, denoted $A = B$, iff $A \subseteq B$ and $B \subseteq A$.

Application: $\{3k+2 : k \in \mathbb{Z}\} = \{3m-1 : m \in \mathbb{Z}\}$.

$\{\dots, -4, -1, \underset{k=0}{2}, \underset{k=1}{5}, \underset{k=2}{7}, \dots\}$ $\{\dots, -4, -1, \underset{m=0}{2}, 5, 7, \dots\}$

① (e.g. $\{3k+2 : k \in \mathbb{Z}\} \cap \mathbb{Z}_{even} \neq \phi$ contains $-4, 10, 16 \dots$)

② Project 5.3 in textbook

Sol^n: We show both LHS ⊆ RHS and RHS ⊆ LHS.

$\boxed{\subseteq}$ $x \in \{3k+2 : k \in \mathbb{Z}\}$, to show that $x \in \{3m-1 : m \in \mathbb{Z}\}$ we need to give $m \in \mathbb{Z}$ s.t. $x = 3m-1$. We know $x = 3k+2$ for some $k \in \mathbb{Z}$. So choose $m := k+1$ because

$$3k+2 \overset{?}{=} 3m-1 \ ? \iff 3k+2 = 3(k+1) - 1 \iff 3k+2 = 3k+3-1 \text{ true} \checkmark.$$

Conversely, for $\boxed{\supseteq}$ we start with $x = 3m-1$, we want $x = 3k+2$.

Choose $k := m-1$ and then $3m-1 = 3k+2$, as needed. ∎

§ 2. De Morgan's Laws:

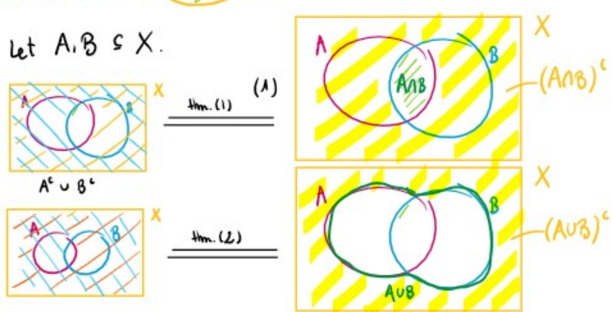
Def: Let $A \subseteq X$, the complement of A in X, denoted A^c, is the set A^c defined by $x \in A^c \iff x \in X$ but $x \notin A$. ∎

Ex: if $A, B \subseteq X$ and $A \subseteq B$, then $B^c \subseteq A^c$.

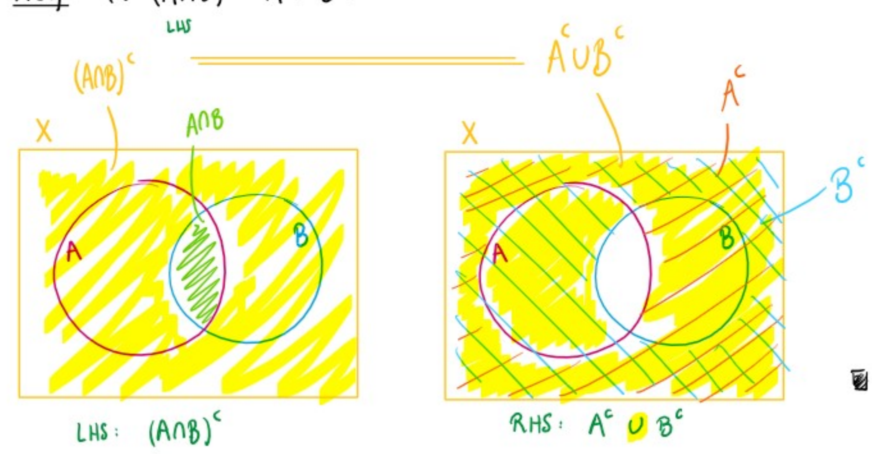
Thm. 5.15 (De Morgan's Laws) Let $A, B \subseteq X$.

(1) $(A \cap B)^c = A^c \cup B^c$.

(2) $(A \cup B)^c = A^c \cap B^c$.



Proof: (1) $\underset{LHS}{(A \cap B)^c} = A^c \cup B^c$.



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