University of California Davis **Name (Print):** ____________________

Differential Equations MAT 108 **Student ID (Print):** ____________________

Midterm Examination **October 30 2020**

Time Limit: 50 Minutes

This examination document contains 5 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Write in your full name and student ID** on the top of every page. The solutions **must be submitted to Gradescope by 10:00am**. The Gradescope window *will* close sharply at 10:00am.

You may *not* use your books, notes, the Internet, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this**: state clearly what the result says, and explain why it may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit. For each solution, make sure to **show all of your work**.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

The table to the right shows the point distribution for the four problems. The highest score is 100.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Show that there are infinitely primes of the form $3k + 2$, for $k \in \mathbb{N}$.

2. (25 points) Solve the following two parts:

   (a) (15 points) Show that for all $n \in \mathbb{N}$ the following inequality holds:

$$2^{n-1} \leq n!$$

   (b) (10 points) Prove that for all $n \in \mathbb{N}$ the following equality holds:

$$\sum_{k=0}^{n} \binom{n}{k} 8^k = 9^n.$$

3. (25 points) Suppose (x_n), $n \in \mathbb{N}$, is a sequence that satisfies the recursion

$$x_{n+1} = x_n + 12x_{n-1}, \quad \text{with } x_1 = 1 \text{ and } x_2 = 25.$$

(a) (10 points) Write down the terms x_1, x_2, x_3, x_4, x_5 and x_6.
(If need be, you may write a product such as $2 \cdot 3$ without multiplying it to $2 \cdot 3 = 6$.)

(b) (15 points) Find a closed formula for the nth term x_n.
(Show all of your work.)

4. (25 points) Solve the following two problems:

   (a) (10 points) Find the last digit of 19^{31}.
   (As always, show all of your work.)

   (b) (15 points) Show that there do not exist two integers $x, y \in \mathbb{Z}$ such that

$$x^2 + 4x + 1 = 4y^2.$$