University of California Davis **Name (Print):** ____________________
Differential Equations MAT 108 **Student ID (Print):** ____________________

Mock Midterm Examination **October 28 2020**
Time Limit: 50 Minutes

This examination document contains 5 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, the Internet, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Show that the following inequalities hold:

   (a) (15 points) Prove that

   $$\sum_{k=1}^{n} \frac{1}{\sqrt{k}} < 2\sqrt{n}, \quad \forall n \in \mathbb{N}.$$

   (b) (10 points) For $n \geq 6$ and $n \in \mathbb{N}$, show that

   $$5n + 5 \leq n^2.$$

2. (25 points) Solve the following two parts:

   (a) (10 points) Consider the sequence $(x_n)_n \in \mathbb{N}$ given by the recursion

   $$x_{n+1} = x_n + (n-1), \quad x_1 = 19.$$

   Find x_{2020}.

   (b) (15 points) Consider the sequence $(x_n)_n$, $n \in \mathbb{N} \cup \{0\}$ defined recursively as

   $$x_n = 7x_{n-1} - 10x_{n-2}, \quad x_0 = 2, x_1 = 3.$$

   Find a closed formula for x_n.

3. (25 points) Solve the following two parts:

   (a) (10 points) Show that the coefficient in front of x^4y^{19} in $(x+y)^{23}$ is 8855.

   (b) (15 points) Consider the expression $(x+y)^n$, show that the coefficient in front of x^ky^{n-k} is the same as the coefficient in front of $x^{n-k}y^k$.

4. (25 points) Solve the following two problems:

   (a) (15 points) Show that there exists no integers $x, y \in \mathbb{Z}$ such that

$$4x^3 - 7y^3 = 2003.$$

   (b) (10 points) Show that the last two digits of 62^{48} are 96.