OHNov23

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Prob. 5(b)

x_n convergent to L $\iff$ $\forall \varepsilon_1 > 0$ $|x_n - L| < \varepsilon_1$ for $n \gg 1$ (*)

y_n convergent to M $\iff$ $\forall \varepsilon_2 > 0$ $|y_n - M| < \varepsilon_2$ for $n \gg 1$ (**)

we have

we want $x_n + y_n$ converges to $L+M$, i.e. we want to

$$\boxed{\forall \varepsilon > 0 \quad |(x_n+y_n) - (L+M)| < \varepsilon}$$

$\varepsilon_1 \overset{?}{=} \varepsilon/2$ check

$\varepsilon_2 \overset{?}{=} \varepsilon/2$

$$|(x_n - L) + (y_n - M)| \leq |x_n - L| + |y_n - M| \underset{(**)}{\overset{}{\underset{(*)}{<}}} \varepsilon/2 + \varepsilon/2 = \varepsilon \quad ■$$

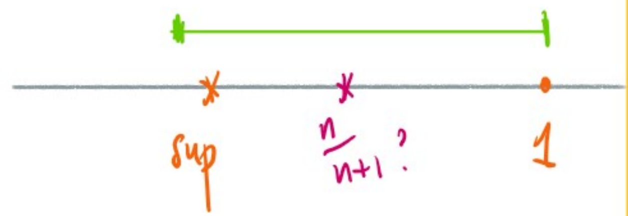
triangle inequality $\leftarrow$ $|a+b| \leq |a| + |b|$

PSet. P4: $\sup(S) = 1$: By contradiction $\sup(S) \neq 1$.

Since 1 is upper bound $\left(\text{b/c } \frac{n}{n+1} \leq 1\right)$ we must have $\sup < 1$.

Now choose $\varepsilon = 1 - \sup > 0$. Then we want that $\exists n \in \mathbb{N}$ s.t.

$$1 - \varepsilon = \sup < \frac{n}{1+n} < 1$$



$\Updownarrow$

$$1 - \varepsilon < \frac{n}{n+1} < 1 \iff 1 - \varepsilon < \frac{1}{1 + \frac{1}{n}} < 1.$$

always true!

$\frac{1}{n} \to 0$

true iff

always true!

true iff

$$1+\frac{1}{n} < \frac{1}{1-\varepsilon} \iff \frac{1}{n} < \frac{1}{1-\varepsilon} - 1. \quad > 0$$

true by Prop. 10.4

■

PSet 5. Prob 6: (b) $x_n = \frac{(-1)^n}{n}$ converges (to 0).

Sol^n: we'll show $L=0$, so we want $\forall \varepsilon > 0$

$$|x_n - 0| < \varepsilon \quad \text{for } n \gg 1.$$

$x_n = \frac{(-1)^n}{n}$

true by Prop. 10.4!

i.e. $\left|\frac{(-1)^n}{n} - 0\right| < \varepsilon \iff \left|\frac{1}{n}\right| < \varepsilon \iff \frac{1}{n} < \varepsilon$ for $n \gg 1$. ■

Fermat's little thm: Let p be a prime.

Then $a^p \equiv a \mod p$

why does it not work if p not prime?

Sol^n: We want $a^p - a \equiv 0 \mod p$,

i.e. $p \mid a^p - a$, $\forall a \in \mathbb{N}$.

(i) if $a \equiv 0 \mod p$, then $p \mid a$, so $p \mid a^p$,

and then $\left.\begin{array}{l} a \equiv 0 \ (p) \\ a^p \equiv 0 \ (p) \end{array}\right\} \ a^p \equiv a \ (p)$.

(ii) if $a \not\equiv 0$, then we show $a^{p-1} - 1 \equiv 0 \mod p$.

↳ this part uses binomial thm:

$$(a+b)^p = \sum_{i=0}^{p} \binom{p}{i} a^i b^{p-i}$$

ONLY if p prime

$$= a^p + b^p.$$

e.g. $4 \nmid \binom{4}{2} = 6$.

if p prime and $1 \leq i \leq p-1$

then $p \mid \binom{p}{i}$.

NOT true unless p is prime!