OH Mon Oct 12

Monday, October 12, 2020 3:13 PM

Prob. 1. Show $\nexists\, n, m \in \mathbb{Z}$ s.t. $n^4 = 4m + 2$.

RHS

Sol^n: (Note that RHS is always (∀m ∈ ℤ))

$\quad\quad\quad\quad\quad\; = 4m+2$

By contradiction, suppose that $\exists\, n, m \in \mathbb{Z}$ with $n^4 = 4m + 2$.

Since $4m+2$ is even, it must be that n^4 is even.

In fact, n^4 being even implies $n \in \mathbb{Z}$ is even.

If n is even, then n^4 is divisible 2^4. In particular $4 \mid n^4$.

Now

$$\underbrace{n^4}_{4|} - \underbrace{4m}_{4|} = 2, \text{ so } 4 \mid \text{RHS}, \text{ i.e. } 4 \mid 2,$$

$4 \mid$ LHS

this is false, so a contradiction. ☒

Prob. 2: Show $\exists$ ∞'ly many primes.

Sol^n: By contradiction, assume $\exists$ finitely many primes $\{p_1, p_2, \ldots, p_N\}$.

Consider the number $\mathcal{P}$:

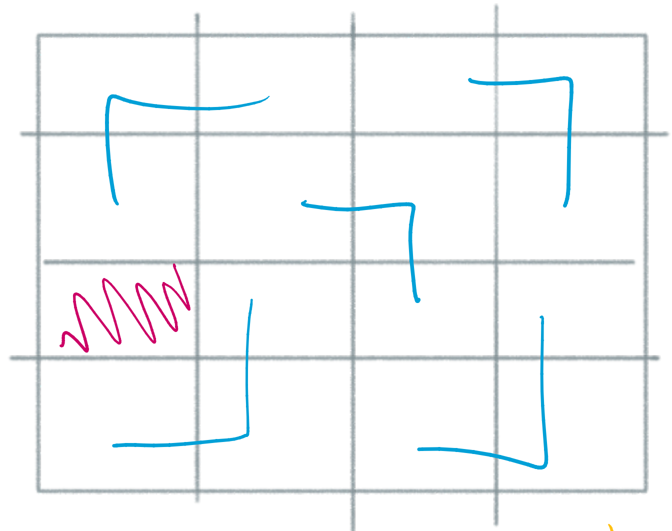
$$\mathcal{P} := p_1 \cdots p_N + 1.$$

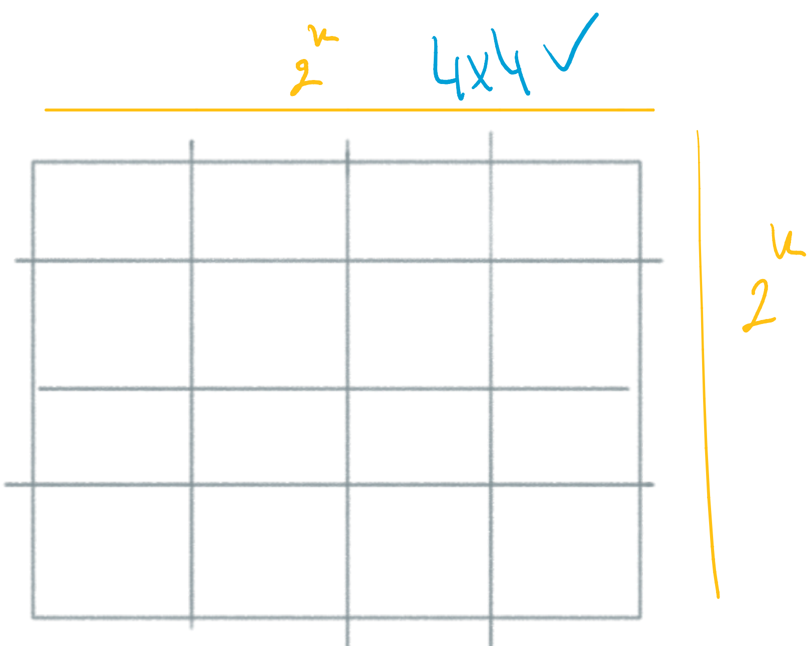
(A) First, $\mathcal{P}$ cannot be prime. Indeed if $\mathcal{P}$ was prime, it'd have to be in the list, so $\mathcal{P} = p_i$ for some i, $1 \leq i \leq N$. But $\mathcal{P} \neq p_i$ by construction, as $\mathcal{P} > p_i$.

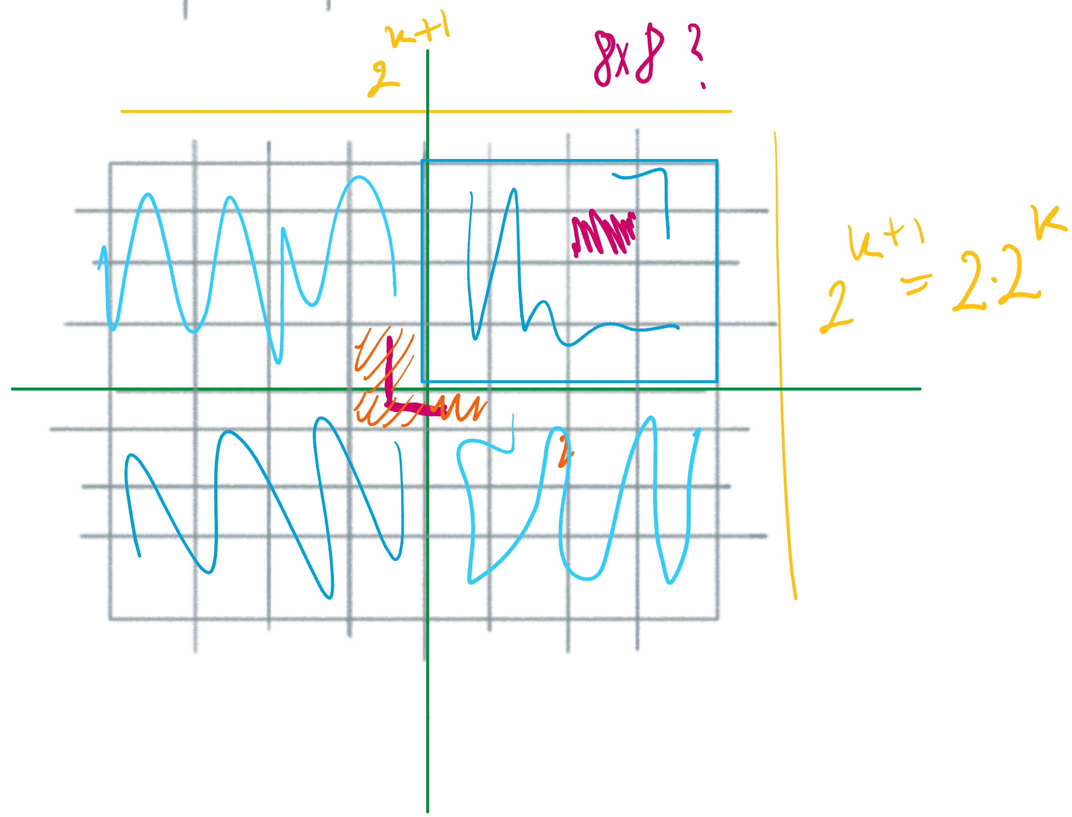
(B) Second, $\mathcal{P}$ is not divisible by p_i. ← This is true as follows: if $p_i \mid \mathcal{P}$ then

note $p_i \mid p_1 p_2 \cdots p_N$ ⟶ $p_i \mid \mathcal{P} - (p_1 p_2 \cdots p_i \cdots p_N)$; but that means $p_i \mid \mathcal{P} - p_1 \cdots p_N = 1$, so $p_i \mid 1$, not possible.

(A) contradicts (B), as any $\mathcal{P}$ not prime must be divisible by a prime, so $\exists p_i \mid \mathcal{P}$. ∎







assume that you can solve for a $2^k \times 2^k$ grid

Solve for

$2^n \times 2^n$ grid