OH Wed Oct 14

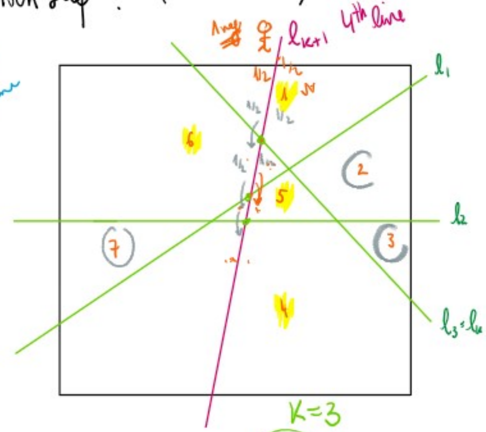
Wednesday, October 14, 2020 4:04 PM

(·) Prob. 3 see Prob 2. (·) Prob. 6 (induction) (·) Prob. 5

Q1: Problem 7: how to do the induction step? (base case $n=1$)

Suppose that k lines divide $\mathbb{R}^2$ ← real plane in $\frac{k^2+k+2}{2}$.

Suppose that the $(k+1)^{st}$ line ℓ_{k+1} is the thrown into $\mathbb{R}^2$:



$K=3$

$1+K$

$$\frac{n(n+1)}{2}+1 = \binom{n}{2}+1 = \left(\sum_{k=1}^{n} k\right)+1$$

R^2

(i) Some regions get divided by 2 and some do not.

How many regions get divided by 2?

$k+1$ ← initially ℓ_{k+1} divides one region in 2 then you change regions every time you hit a line $\ell_1, \dots, \ell_k$.

Since there are k lines, we get $1 + k$. ← initial region ← new region as we cross $\ell_1, \ell_2, \dots, \ell_k$

(ii) in brief: $\frac{k^2+k+2}{2} + (k+1) = \frac{k^2+k+2+(k+1)2}{2} \overset{?}{=} \frac{(k+1)^2+(k+1)+2}{2}$

check!

\# regions w/ k lines $\overset{(*)}{=} \frac{k^2+k+2}{2}$ by induction assumption.

\# regions w/ $(k+1)$-lines $\overset{\text{we want}}{=} \frac{(k+1)^2+(k+1)+2}{2}$

|| equal! (*)

$$\left(\begin{matrix}\text{\# region w/}\\ k \text{ lines}\end{matrix}\right) + (k+1) = \boxed{\frac{k^2+k+2}{2} + (k+1)}$$

Prob. 3: $\exists$ ∞'ly many primes of the form $4k+3$.

Sol^n: By contradiction, assume $\exists$ finitely many primes of the $4k+3$.

Sol: By contradiction, assume $\exists$ finitely many p ...

Call them $\{p_1, p_2, \dots, p_N\}$. Consider $P := 4(p_1 p_2 \cdots p_N) - 1$.

↳ only primes of form $4k+3$

(i) $p_i \nmid P$, because if it did $p_i \mid 1$. Now, since $P = 4k+3$, at least $\exists p_i$ of the form $4k+3$ dividing it.

(ii) $p_i \neq P$ as $p_i \lneq P$. $\boxed{\#}$

Prob. 5: (i) $1+2+3+\dots+k = \dfrac{k\cdot(k+1)}{2}$

By induction, (1) the base case is $k=1$: $1 = \dfrac{1\cdot(1+1)}{2} = \dfrac{1\cdot 2}{2} = 1$ ✓.

(2) Induction step: we assume $\boxed{(*)\ 1+2+3+\dots+k = \dfrac{k(k+1)}{2}}$ (k case)

We want $\underbrace{1+2+3+\dots+k}_{(*)\,=} + (k+1) \overset{??}{=} \dfrac{(k+1)(k+2)}{2} = \dfrac{k^2+k+2k+2}{2}$ ✓ true!

$$\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)+2(k+1)}{2} = \frac{k^2+k+2k+2}{2} \quad \boxed{\#}$$

Prob. 6:

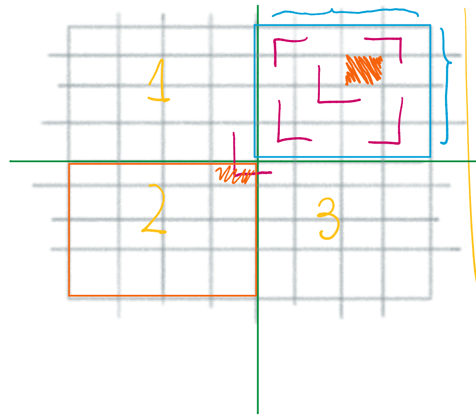
Base case:

Ind. step:

Step:

2^k

assume that you can solve for a $2^k \times 2^k$ grid



$2^{k+1} = 2 \cdot 2^k$