OH Mon Oct 5

Monday, October 5, 2020 4:05 PM

- Working through Prop w/ axioms (V.3 PSet 1) Prob 4.

- Q6 PSet 1
- Prop 2.13
- Q1
- Q5
- Q2
- Prop 1.10

In the Spirit of Prob 4 PSet 1:

$\# S1$: "$\exists a \in \mathbb{Z}$ s.t. $\forall n \in \mathbb{Z}$, $a+3n=7$."

$\# S2$: "$\forall n \in \mathbb{Z}$ $\exists a \in \mathbb{Z}$ s.t. $a+3n=7$."

Analysis of S1: $a \in \mathbb{Z}$ is chosen first (so indep. of $n \in \mathbb{Z}$).

How to argue S1 false? S1 should be true for all $n \in \mathbb{Z}$.

So $a \in \mathbb{Z}$ must satisfy $a+3n=7$ for all $n \in \mathbb{Z}$.

In particular, for $n=48$ and also for $n=42$. Thus it must be that

$a+3\cdot 48=7$ and $a+3\cdot 42=7$.

This means $a+144=7$ and $a+132=7$.

The first eq. implies $a=-137$ and $a=-125$, since $125 \neq 137$, this is a contradiction and S1 is false. $\square$

Analysis of S2: we first choose $n \in \mathbb{N}$ (which can be arbitrary) and then we want $a \in \mathbb{Z}$ (a can depend on n!) s.t. $a+3n=7$.

So, let's prove S2 is true: we need to show $a \in \mathbb{Z}$ exists.

For that, given $n \in \mathbb{Z}$ we choose

$$a := 7-3n \in \mathbb{Z}.$$

This a satisfies

$a+3n = (7-3n)+3n = 7$ as desired. $\blacksquare$

⚠ In Prob. 4: it's not $\mathbb{Z}$ but $\mathbb{N}$.

(Prop. 2.13) $\mathbb{N} = \{n \in \mathbb{Z} : n > 0\}$. ← in order to prove equality $X=Y$ of sets one proves $X \subseteq Y$ and $Y \subseteq X$.

Proof: $\supseteq$ | Prove $\{n \in \mathbb{Z} : n>0\} \subseteq \mathbb{N}$.

For $x \in \{n \in \mathbb{Z} : n>0\}$, we need to conclude $x \in \mathbb{N}$.

By definition, $x>0$ iff $x-0 \in \mathbb{N}$ (def of 2.2), now $x-0=x$, so $x \in \mathbb{N}$ $\blacksquare$

$\subseteq$ | For $x \in \mathbb{N}$ then $x \in \{n \in \mathbb{Z} : n>0\}$.

By definition, $x = x-0$. Since $x-0=x \in \mathbb{N}$, we conclude $x>0$ $\blacksquare$

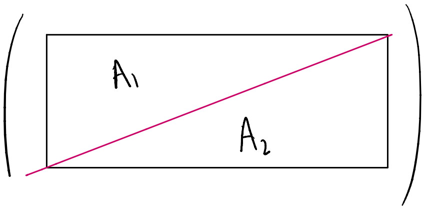
Q1 of PSet 1 (c) Statement is about $a=3, b=4, c=5$ being the only solutions. THIS is FALSE.

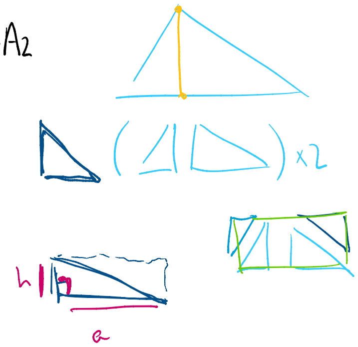
Why? There are other solutions to $a^2+b^2=c^2$ with $a,b,c \in \mathbb{Z}$.

Choose $a=\overset{6}{2\cdot 3}$, $b=\overset{8}{2\cdot 4}$, $c=\overset{10}{2\cdot 5}$: $2^2\cdot 3^2+2^2\cdot 4^2 = 2^2\cdot 5^2$

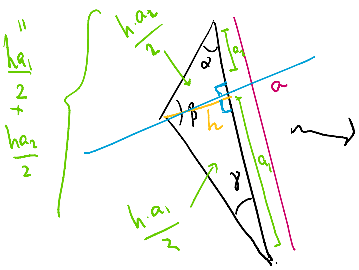
$\Leftrightarrow \cancel{2^2}\cdot(3^2+4^2) = \cancel{2^2}\cdot 5^2 \Leftrightarrow 3^2+4^2=5^2$ true. $\blacksquare$

$\Leftrightarrow \cancel{2^2}\cdot(3^2+4^2)=\cancel{2^2}\cdot 5^2 \Leftrightarrow 3^2+4^2=5^2$ true.

Q5: Area  $= A_1+A_2$



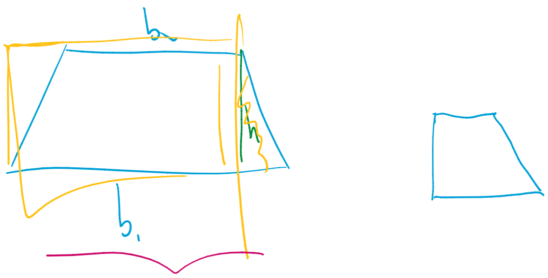
$\frac{h(a_1+a_2)}{2}=\frac{h\cdot a}{2}$ if $A_1=A_2$ then Area(□) $=2A_1=2A_2$.



double it to cook up rectangle

$a\cdot h = 2\cdot$ Tria

Area tr $= \frac{a\cdot h}{2}$.



Zach's conj: $k=a+b$, $a,b\in\mathbb{N}$ then $k \mid a^{2n+1}+b^{2n+1}$ for all $n\in\mathbb{N}$. (?)

Possible proof: By induction on $n\in\mathbb{N}$,

Base case: $n=1$, for $n=1$ this $k \mid a^3+b^3$. Since we can write

$$a^3+b^3=(a+b)\cdot(a^2-ab+b^2)$$

$$k=a+b \mid k\cdot(a^2-ab+b^2)=a^3+b^3.$$

Induction step: assume $k \mid a^{2n+1}+b^{2n+1}$, we want $k \mid a^{2(n+1)+1}+b^{2(n+1)+1}$,

For that write

$$a^{2n+3}+b^{2n+3}=\left(a^{2n+1}+b^{2n+1}\right)\left(\text{find this term!}\right)$$

$$\underbrace{a^{2n+3}+b^{2n+3}}_{= a^{2n+1}\cdot a^{2}+b^{2n+1}\cdot b^{2}} = \Big(\underbrace{a^{2n+1}+b^{2n+1}}_{\text{K divides by hyp.}}\Big)\Big(\ \text{term!}\ \Big). \qquad \blacksquare$$