OH Wed Oct 7

Wednesday, October 7, 2020 4:04 PM

Wishlist:

- $\sum_{k=1}^{n}(2k-1)=n^2$ ✓
- Prob 1.(d) ✓
- Prop 1.12 vs. 1.13 ✓
- PSet 1 #4

(i) Show that $\sum_{k=1}^{n}(2k-1) = n^2$. (odd $\mathbb{N}$)

Proof: By induction, the statement is $P(n) =$ " $\sum_{k=1}^{n}(2k-1)=n^2$ ".

Base Case: Need to check $P(1)$ true. $P(1)$ is $\sum_{k=1}^{1}(2\cdot k-1) = 2\cdot 1-1 = 1 = n^2$ for $n=1$.

Induction: Suppose $\sum_{k=1}^{n} 2k-1 = n^2$, i.e. $\boxed{1+3+5+\dots+(2n-1) = n^2 \quad (*)}$ inductive hyp. — assume true

we want $\sum_{k=1}^{n+1} 2k-1 = (n+1)^2$, i.e. $\boxed{\underset{k=1}{1}+\underset{k=2}{3}+\underset{k=3}{5}+\dots+\underset{k=n}{(2n-1)}+\underset{k=n+1}{(2n+1)} \overset{???}{=} (n+1)^2.}$ — $P(n+1)$ true?

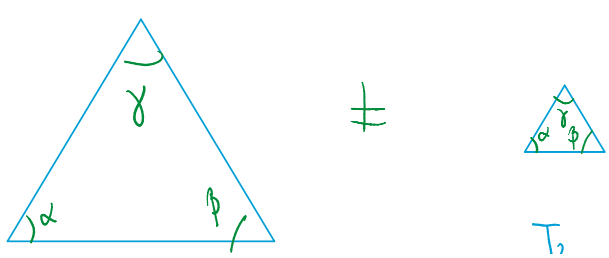
Now,

$\boxed{1+3+5+\dots+(2n-1)+(2n+1) = (n+1)^2} \iff \underbrace{1+3+5+\dots+(2n-1)}_{\text{LHS of } (*)}+(2n+1) = n^2+2n+1$

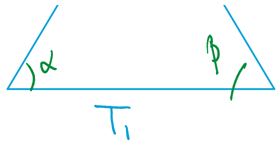
$\iff$ (we use the $(*)$ inductive hyp.) $n^2+(2n+1) = n^2+2n+1$, which is true. ∎

1.(d) Let $\alpha,\beta,\gamma \in [0,2\pi)$. There exists a unique triangle in the plane s.t. the interior angles are α,β,γ.

Sol^n: This is false. One counter example is by using the fact that 2 different triangles T_1, T_2 might have same interior angles. For example:



but same α,β,γ



T_2

A different counterexample: the angles α, β, γ must sum to $\alpha+\beta+\gamma = \pi$. $(=180)$

So choosing $\alpha = \pi/2$, $\beta = \pi/2$, $\gamma = \pi/2$

there cannot be a triangle with these. ∎

Prop. 1.12 & 1.13: $\boxed{\text{uniqueness of } 0}$ Suppose $a \in \mathbb{Z}$ s.t. $\forall m \in \mathbb{Z}$ $m + a = m$.

then $\underset{\substack{\| \\ a+(m-m) \\ \| \leftarrow A.1.4 \\ a}}{m+a} - m = \underbrace{m-m}_{\substack{\| \\ 0}} \leftarrow \underline{A.1.4}$, i.e. $a = 0$.

Prop. 1.12: x s.t. $\forall m$ $m+x=m$, then $\underbrace{-m+m}_{\substack{\| \\ 0}}+x = \underbrace{-m+m}_{\substack{\| \\ 0}}$ $\Rightarrow x=0$.

$\underline{A.1.4}$

Prop 1.12: let $x \in \mathbb{Z}$. If $\underline{\forall m \quad m+x = m}$ (assumption), then $x = 0$.

Prop 1.13: let $x \in \mathbb{Z}$. If $\underline{\exists m \text{ s.t. } m+x=m}$, then $x=0$.

assumption

Proof Prop. 1.13: Since $m+x=m$, then $-m+m+x = -m+m$

and by A.1.4 $(-m)+m=0$, so $0+x=0$, hence $x=0$ by A.1.2. ∎

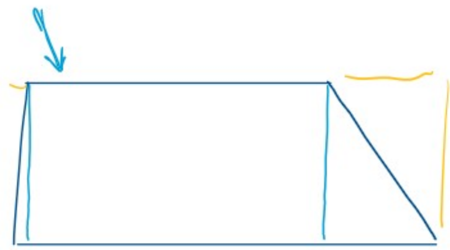
- Negate Statement 1
- Prob. 6

S1: "$\exists a \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}$ we have $n + a = 7$." ← false

negating: "$\nexists a \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}$ we have $n + a = 7$." ← tr

There does not exist $a \in \mathbb{N}$ s.t.

Prob 6.



Prob. (4). $\forall k \in \mathbb{N},\ 4 \mid k^4 - 6k^3 + 11k^2 - 6k$.

Sol^n: By induction, the base case $4 \mid 1^4 - 6\cdot 1^3 + 11\cdot 1^2 - 6 = 0$, which is true.

The induction step: assume $4 \mid \boxed{k^4 - 6k^3 + 11k^2 - 6k}$, we want $4 \mid \underbrace{(k+1)^4 - 6(k+1)^3 + 11(k+1)^2 - 6(k+1)}_{\text{expand this}}$.