Lecture 1

Tuesday, September 29, 2020 12:05 PM

INTRO TO MAT-265

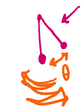
- field := section of a principal G-bundle (or vector bundle)
  ↳ functions, maps $X \to Y$, vector fields, 1-form, etc.
  ↳ electrodynamics
- dynamics := there is a rule that governs time evolution.
- $\boxed{\text{mechanics}}$ v. field theory ← ∄ def of field thy

study $\underset{\text{time}}{\mathbb{R}} \longrightarrow \underset{\text{conf. space}}{M}$

study $X \longrightarrow M$ where X more general

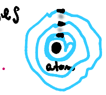
§ 1. Classical Mechanics vs. Quantum Mechanics

(a) Continuum of energies.



(b) Particle: point modelled by
$\gamma : [a,b] \longrightarrow M$, ($t$)
$\gamma(t)$ = position of particle at time t.

(a) Quantized quantities
↳ Energies $E_n \in \mathbb{N}$ e.g.



(b) Many particles have properties from a "classical point particle" and also from a "wave".
↳ double-slit experiment
& photo-electric effect.

§ 2. Classical Mechanics : starts with I. Newton's $x''m = F$.

(acceleration, mass, force)

(Friday)
(i) Lagrange formulation : on Pset!
"CALCULUS OF VARIATIONS"

math

The dynamics, i.e. the choice of paths $\gamma : [a,b] \longrightarrow M$ is given by minimizing the "action"

$$S(\gamma) := \int_a^b \underbrace{L(t, \gamma(t), \dot{\gamma}(t))}_{\text{Lagrangian}}\, dt .$$

∞'dimensional

action $\to S : \Omega \longrightarrow \mathbb{R}$

↳ Ω is a space of functions $[a,b] \to M$

TM

some cases

(next week)
(ii) Hamiltonian formulation

The input is the Hamiltonian
$H : T^*M \longrightarrow \mathbb{R}$.

Ex: $M = \mathbb{R}^n_q$, $T^*M = \mathbb{R}^n_q \times \mathbb{R}^n_p$,

$$H(q,p) = \tfrac{1}{2}(p^2 + q^2)$$

(kinetic, potential)

"SYMPLECTIC GEOMETRY" ⇝ Hamilt. eq^{ns}.

§ 3. Lagrangian Formulation :

$f : M \to \mathbb{R}$ function, df 1-form
$\mathrm{crit}(f) = \{ p \in M : df(p) = 0 \}$.

Next time : $\Omega = \{ \gamma : [a,b] \to \mathbb{R}$ smooth $\}$ domain,

$S : \Omega \longrightarrow \mathbb{R}$ action functional

$\mathrm{Crit}(S) := \{ \gamma \in \Omega$ s.t. $\delta S(\gamma) = 0 \}$ ← γ crit. point ⇏ minimum (need a second derivative test)

The case at hand is $S(\gamma) = \int_a^b L(t, \gamma(t), \dot{\gamma}(t))\, dt$,

how do you compute $\delta S(\gamma)$?