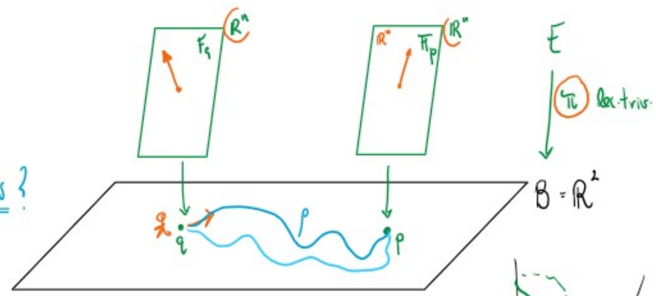
OH Nov 25

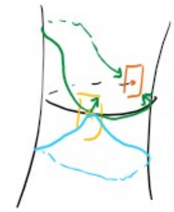
Wednesday, November 25, 2020 2:56 PM

Parallel transport:



How to identify fibers?

(1) Choose a path ρ between p & q.



(2) A connection is the information that identifies $F_q \overset{\rho}{\cong} F_p$.

In general, different ρ's get you different isom.

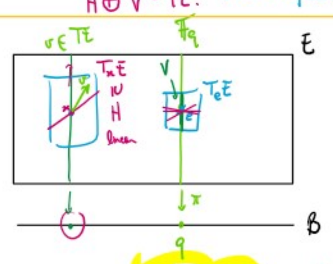
(i) for each ρ a path in B between q, p we get iso: $F_q \cong F_p$

(*)(ii) (Ehresmann) Consider $V := \ker d\pi$.
A connection $H \subseteq TE$ subbundle s.t. $H \oplus V = TE$. (i.e. a complement to V)

⇝ (iii) A $\mathfrak{g}$-valued 1-form $\Omega^1(E, \mathfrak{g})$ where $\mathfrak{g} = \mathrm{Lie}(G)$, G struct. gp of E.

gauge gp.

$A \in \Omega^1(E, \mathfrak{g})$



$$TE \supseteq \ker d\pi$$
$$\downarrow d\pi$$
$$TB$$

if E v.b. w/ $G = GL_n$ a connection is just a 1-form with coeff. in matrices $\mathfrak{g} = \mathrm{End}(\mathbb{R}^n)$.

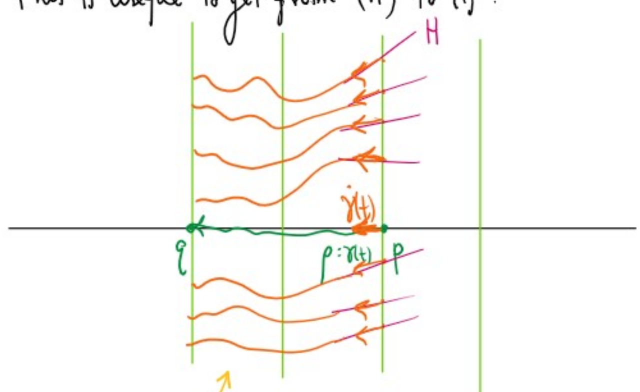
$$\left(0 \to \underset{\ker d\pi}{V} \hookrightarrow TE \xrightarrow{d\pi} TB \to 0\right)$$

splitting of s.e.s. ⇔ connection

We can use (iii), or (ii) equiv, to build a covariant derivative:

(locally): $d_A := d + A$.

(*) This is useful to get from (ii) to (i):



this identification via flow of lift is (i), it's called THE PARALLEL TRANSP. of H.

(*) From (iii) to (i): you solve the ODE

$$\boxed{d_A \dot\gamma = 0}$$

Parallel transp = Integrated version of a connection

(*) From (iii) to (ii): given d_A cov. deriv.,

call a section s of $\pi: E \to B$ <u>flat</u> if

$$d_A s = 0$$

$\rightsquigarrow$ H is given by tg. space to graphs of flat sections