def: the variable $x$ (centered at 0) is an expression

$$A(x) := \sum_{n=0}^{\infty} a_n x^n$$

where $a_n \in IR$

$$= a_0 + a_1 x + a_2 x^2 + \ldots$$

why do we do this? approximate functions at a point thru polynomials / $x$ values determine convergence

taylor's thm (on wed) @ $x_\approx 0$

$$A(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

(everything is green / converges)

**thm 18:** suppose $\sum_{n=0}^{\infty} a_n x^n$ is power series then:

1) if $A(x)$ converges for $x=c$, $c \in IR$, then it converges for $|x| < c$

2) if $A(x)$ diverges @ $x=d$, $d \in IR$, then it diverges for $|x| > d$

remark: it can not ...

**ex 1)** geometric series: $A(x) = \sum_{n=0}^{\infty} x^n (a_n = 1)$

converges for $|x| < 1$, diverges for $|x| \geq 1$

**corollary of thm 18:** given $A(x)$ power series, we have:

1) $\exists R$ such that $|x| < R$ $|x| < R$ $R = x$ $|x| > R$ $|x| > R$
corollary of thm 18: given \( A(x) \) power series, we have:

1) \( \exists R \) such that
   \[ 1 \times 1 > R \quad \frac{1}{1-x} < R \quad \frac{R}{1-x} > R \]
   what is \( R \)?
   name: radius of convergence

2) all is green:
   \[ R = \infty \]

3) all is red except \( x = 0 \):
   \[ R = 0 \]

caution: nothing said for \( x = \pm R \) (endpoints) → study those directly (by plugging \( x = R, x = -R \) and see)