Recall: \( \mathbf{r}(t) \) = trajectory
\[ \dot{\mathbf{r}}(t) = \mathbf{v}(t) \] = velocity
\[ \ddot{\mathbf{r}}(t) = \mathbf{a}(t) \] = acceleration.

- For each value of \( t \), we get a vector.

Question:

1. What if we know \( \mathbf{r}(0) \) and velocity \( \mathbf{v}(t) \) at all times? Find \( \mathbf{r}(t) \) (when integrate once -> need 1 constant \( \mathbf{r}(0) \)).

2. If we know \( \mathbf{r}(0) \) and \( \mathbf{v}(t) \), can we find \( \mathbf{r}(t) \) if \( \mathbf{v}(t) \) known? (When integrate twice -> need 2 constants \( \mathbf{r}(0) \) & \( \mathbf{r}(0) \)).

Example: particle starts @ \( \mathbf{r}(0) = (1, 0, 0) \) & has velocity \( \mathbf{v}(t) = (-\sin(t), \cos(t), \dot{c}) \). Find \( \mathbf{r}(4\pi) \) = position @ time \( t = 4\pi \).

Solution: since \( \dot{\mathbf{v}}(t) = \ddot{\mathbf{r}}(t) \), we have
\[ \mathbf{r}(t) = \int \ddot{\mathbf{r}}(t) + \mathbf{c} \] = indefinite integral
vector \( \mathbf{c} \) = \( \langle c_1, c_2, c_3 \rangle \)

\[ \int \ddot{\mathbf{r}}(t) \] = \( \langle -\sin(t), \cos(t), \dot{c} \rangle \)

\[ \mathbf{r}(t) = \int \langle -\sin(t), \cos(t), \dot{c} \rangle \, dt = \langle -\cos(t), \sin(t), \dot{c} + c_1 \rangle + \langle c_1, c_2, c_3 \rangle \]

\[ \mathbf{r}(t) = \langle \cos(t) + c_1, \sin(t) + c_2, \frac{\dot{c}}{a} + c_3 \rangle \]

To find \( c_1, c_2, c_3 \), we use \( \mathbf{r}(0) = (1, 0, 0) \)

It gives:
\[ \mathbf{r}(0) = \langle \cos(0) + c_1, \sin(0) + c_2, \frac{\dot{c}}{a} + c_3 \rangle = (1, 0, 0) \]

Thus \( c_1 = 0, c_2 = 0, \frac{\dot{c}}{a} = 0 \) \( \leftarrow \) in general not always 0
\[ \mathbf{r}(t) = \langle \cos(t), \sin(t), \frac{\dot{c}}{a} \rangle \]

\( \Theta + 4\pi \):
\[ \mathbf{r}(4\pi) = \langle \cos(4\pi), \sin(4\pi), \frac{\dot{c}}{a} \rangle = \langle 1, 0, \frac{10\pi^2}{a} \rangle = \langle 1, 0, 8\pi^2 \rangle \]