Spheres & Inequalities

**Chapter 12.1 Page 1**

Sphere of radius $R$ & center $(c_1, c_2, c_3)$:

\[(x - c_1)^2 + (y - c_2)^2 + (z - c_3)^2 = R^2\]

*all points $(x, y, z)$ @ distance $R$ from center*

ex 1) draw $S = \{x^2 + y^2 + z^2 = 9\}$ & decide if $(1, 0, 0)$, $(2, 1, 2)$ & $(0, 0, 327)$ belong

- $(1, 0, 0): 1^2 + 0^2 + 0^2 = 1 < 9 \times (\text{inside})$
- $(2, 1, 2): 2^2 + 1^2 + 2^2 = 9 \times (\text{on sphere})$
- $(0, 0, 327): 0^2 + 0^2 + 327^2 = 9 \times (\text{outside})$

Inequalities: geometrically an equation with $\leq$ describes surface separating space into 2 pieces

if given $x, y, z, \geq, \leq$...

1st) analyze equality case $\rightarrow$ get surface & 2 pieces of space

and) sample points in inequality to decide which (st 2) piece

ex a) describe ( & draw) $\{x + y + z < 1\}$

1st) note $\{x + y + z = 1\} = \text{plane}$

\[0 - 5 + 0 < 1 \checkmark\]

so we get back half

\[0 + 5 + 0 < 1 \times\]

Intersections: more than 1 equation $(=, \leq, \geq, >, \leq)$

ex 3) describe & draw points in space such that

$\{x + y + z = 1\}$ and $\{z = 0\}$

\[
\text{plane written as comma plane as comma plane}
\]
ex 3) describe \( \begin{cases} x + y + z = 1, \\ z = 0 \end{cases} \)

intersection is line
thru \((1, 0, 0)\) & \((0, 1, 0)\)

ex 4) draw \( \begin{cases} x^2 + y^2 + z^2 = 9, \\ y = 2 \end{cases} \)

we only distinct y value
\((a, b, c) = (0, 1, 0) \) & contains \((0, b, 0)\)

intersecion