MAT 21C: PRACTICE PROBLEMS LECTURE 10

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the tenth lecture of Part II, delivered May 22 2023. Solutions will be posted within 48h of these problems being posted.

Reminder: the 2nd Newton Law states $F(t) = m \cdot a(t)$, where $F(t)$ is the force vector, m the mass of the particle, and $a(t)$ its acceleration vector.

Problem 1. Suppose that a particle starts at $r(0) = \langle 5, 2, -1\rangle$ and has velocity vector

$$v(t) = \langle t^3, \sin(t), e^{-t}\rangle.$$

(a) Find the trajectory $r(t)$ of the particle.
(b) Where is the particle at $t = 10$ seconds?

Problem 2. Suppose that a particle starts at $r(0) = \langle 0, 0, 1\rangle$ and has velocity vector

$$v(t) = \langle 0, 0, -t\rangle.$$

Find when the particle will hit the origin $(0, 0, 0)$, i.e. find the time t so $r(t) = \langle 0, 0, 0\rangle$.

Problem 3. A pebble of mass $m = 0.1kg$ is dropped from the point $S = (0, 0, 3)$, with initial velocity $v(0) = \langle 0, 0, 0\rangle$. Assume that the force of gravity is $F(t) = \langle 0, 0, -9.81\rangle$.

(a) Find the position of the particle after $t = 15$ seconds.
(b) Does the particle ever pass through the origin $(0, 0, 0)$? If so, at what time t?

(c) Suppose there is increasingly stronger winds blowing west according to

$$\langle 0, 5t, 0\rangle,$$

and thus the force applied to the particle is instead

$$F(t) = \langle 0, 5t, -9.81\rangle.$$

Find the position of the particle after $t = 15$ seconds and show that it will never hit the origin $(0, 0, 0)$.

Problem 4. Consider a particle in space moving with acceleration

$$a(t) = \langle t^2, 3 - \cos(t), 45t + e^t\rangle.$$

Suppose the initial velocity of the particle is $v(0) = \langle 0, 0, 1\rangle$ and its initial position is $r(0) = \langle 2, 0, 3\rangle$.

(a) Find the velocity $v(t)$ of the particle.
(b) Find the speed of the particle at $t = 0$ and at $t = 10$.
(c) Find the position $r(10)$ of particle at $t = 10$.