MAT 21C: PRACTICE PROBLEMS LECTURE 11

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the eleventh lecture of Part II, delivered May 31 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Evaluate the following functions $f(x,y)$ at the indicated points (x,y).

(a) Evaluate $f(x,y) = e^{xy}\cos(x+y)$ at $(0,0)$, $(\pi,0)$, $(0,\pi)$ and (π,π).

(b) Evaluate $f(x,y) = x^2y - x + 3y^2 + 4$ at $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$.

(c) Evaluate $f(x,y) = \sqrt{x+y+\sin(\pi xy)}$ at $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$.

Problem 2. For each of the following functions $f(x,y)$, compute the two partial derivatives $\partial_x f$ and $\partial_y f$.

(a) $f(x,y) = e^{xy}\cos(x+y)$.

(b) $f(x,y) = x^2y - x + 3y^2 + 4$.

(c) $f(x,y) = \sqrt{x+y+\sin(\pi xy)}$.

(d) $f(x,y) = x^7\ln(y^3\cos(x^2)) - e^{xy^2}\sqrt{y}$.

Problem 3. For each of the following functions $f(x,y)$, evaluate the partial derivatives $\partial_x f$ at the indicated points.

(a) For $f(x,y) = e^{xy}\cos(x+y)$, evaluate $\partial_x f$ at $(x,y) = (0,0)$.

(b) For $f(x,y) = x^2y - x + 3y^2 + 4$, evaluate $\partial_x f$ at $(x,y) = (5,1)$.

(c) For $f(x,y) = \sqrt{x+y+\sin(\pi xy)}$, evaluate $\partial_x f$ at $(x,y) = (2,0)$.

(d) For $f(x,y) = x^7\ln(y^3\cos(x^2)) - e^{xy^2}\sqrt{y}$, evaluate $\partial_x f$ at $(x,y) = (\pi,1)$.

Problem 4. For each of the following functions $f(x,y)$, show that $(x,y) = (0,0)$ is the only critical point:

(a) $f(x,y) = x^2 + y^2$.

(b) $f(x,y) = x^2 - y^2$.

(c) $f(x,y) = -x^2 - y^2$.

Problem 5. For each of the following functions $f(x, y)$, find *all* the critical points:

(a) $f(x, y) = (x-5)^2 - (y+7)^2 + 5$.

(b) $f(x, y) = e^{-x^2-y^2}$.

(c) $f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$.

(d) $f(x, y) = x^4 + y^4 - 4xy$.

(e) $f(x, y) = e^{x-y^2-x^3/3}$.

(f) $f(x, y) = x + 3y - 17$.

Hint: The first two functions in $(a), (b)$ *have each only one critical point. The one in* (c) *has four critical points, the one in* (d) *has three and the one in* (e) *has two critical points. Finally, the function in* (f) *has none.*