MAT 21C: PRACTICE PROBLEMS LECTURE 5

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the fifth lecture of Part II, delivered May 10 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Consider the vector $v = \langle -2, 1, -1 \rangle$.

(a) Compute the length $|v|$.
(b) Find the dot product $v \cdot v$.
(c) Verify that, in this case, $|v|^2 = v \cdot v$.
(d) Find the unit length vector $\dfrac{v}{|v|}$.

Problem 2. Consider the vectors $v = \langle 3, -7, 1 \rangle$ and $w = \langle -1, 4, 2 \rangle$.

(a) Find the dot product $u \cdot v$.
(b) Compute the length $|v|$.
(c) Compute the scalar component of u in the direction of v.
(d) Find the projection of u in the direction of v.
(e) What is the angle between u and v?

Problem 3. Consider a vector $v = (v_1, v_2, v_3)$.

(a) Show by direct computation that $|v|^2$ equals $v \cdot v$.
(b) As a second method, use the dot product-angle formula to deduce $|v|^2 = v \cdot v$.
(c) As a third method, justify geometrically in terms of projections why $|v|^2 = v \cdot v$.

Problem 4. Consider the vectors $u = \langle -1, 2, 0 \rangle$ and $v = \langle 3, 2, -5 \rangle$.

(a) Show that $w = \langle 0, 0, 1 \rangle$ is perpendicular to u but not to v.
(b) Show that $w = \langle 1, 1, 1 \rangle$ is perpendicular to v but not to u.
(c) Show that $w = \langle 10, 5, 8 \rangle$ is perpendicular to both v and u.

Problem 5. Consider the vectors $u = \langle -5, 8, 1 \rangle$ and $v = \langle -2, 3, 7 \rangle$.

(a) Find a non-zero vector which is perpendicular to u but not v.
(b) Find a non-zero vector which is perpendicular to v but not u.
(c) Find a non-zero vector which is perpendicular to both u and v.

Problem 6. Consider the plane $\pi = \{ax + by + cz = 0\}$ for some $a, b, c \in \mathbb{R}$. Explain using the dot product why $\langle a, b, c \rangle$ is the perpendicular direction to π.

Hint: The left hand side $ax + by + cz = 0$ of the equation can be writtent as $\langle a, b, c \rangle \cdot \langle x, y, z \rangle$. Also, the endpoint of $\langle x, y, z \rangle$ belong to π if and only if $ax + by + cz = 0$.