Abstract. Practice problems for the seventh lecture of Part II, delivered May 15 2023. Solutions will be posted within 48h of these problems being posted.

Recall the four descriptions of a plane \(\pi\):

1. The plane \(\pi\) is given by three points \(P, Q, R \in \pi\).
2. The plane \(\pi\) is given by one point \(P \in \pi\) and a normal direction \(n = \langle a, b, c \rangle\).
3. The plane \(\pi\) is given by one point \(P \in \pi\) and two vectors \(u, v\) inside of \(\pi\).
4. The plane \(\pi\) is given by an equation \(\pi = \{ax + by + cz = d\}\), where \(a, b, c, d \in \mathbb{R}\) are real numbers.

**Problem 1.** Consider the unique plane \(\pi\) containing the three points \(P = (1, 1, 2), Q = (-2, 3, 0)\) and \(R = (0, -5, 7)\).

(a) Find two vectors \(u, v\) inside of \(\pi\).
(b) Compute a perpendicular direction to \(\pi\).
(c) Find an equation for \(\pi\).

**Problem 2.** Find an equation for the plane through point \((9, 3, -1)\) parallel to the plane \(\{x + y + z = 0\}\).

**Problem 3.** Consider the three planes

\(\pi_1 = \{3x - 5y + 4z = 12\}\)
\(\pi_2 = \{\text{unique plane that contains } (0, 1, 0) \text{ with perpendicular direction } (1, 4, 3)\}\)
\(\pi_3 = \{\text{unique plane that contains } (0, 0, 0) \text{ and vectors } u = (2, 4, 1), v = (2, -5, 12)\}\)

(a) Show that \(\pi_1\) intersects \(\pi_2\) at a line, \(\pi_1\) intersects \(\pi_3\) at a line, and \(\pi_2\) intersects \(\pi_3\) at a line. (That is, these are not parallel to each other.)
(b) Find the directions of each of these lines.

\footnote{It is fine if \(u, v\) are just two vectors in the direction parallel to \(\pi\).}
Problem 4. Consider the two planes

\[ \pi_1 = \{3x + 3y + 3z = 12\} \]
\[ \pi_2 = \{ \text{unique plane that contains (0,0,0) with perpendicular direction } (1,1,1) \} \]

(a) Show that \( \pi_1 \) and \( \pi_2 \) are parallel planes and they are different.

(b) Find a plane \( \pi_3 \) different than \( \pi_1 \) and \( \pi_2 \) but is parallel to both of them.

Problem 5. Consider the plane \( \pi = \{2x + 9y - z = 3\} \).

(a) Find three distinct points \( P, Q, R \in \pi \) that belong to \( \pi \).

(b) Find two vectors \( u, v \) which are parallel to \( \pi \).

(c) Find a plane \( \pi' \) parallel to \( \pi \) but different from it.

(d) Find a plane \( \pi'' \) which intersects \( \pi \) at a line.

Problem 6. Consider the plane \( \pi = \{2x + y - z = 0\} \) and the unique line \( L \) through the origin and the point \( P = (0,1,1) \).

(a) Argue that the point \( P \in \pi \) belongs to the plane \( \pi \).

(b) Justify that the line \( L \) lies inside the plane \( \pi \).

(c) Find a plane \( \pi' \) such that their intersection is the line \( L \).