MAT 21C: PRACTICE PROBLEMS LECTURE 8

PROFESSOR CASALS (SECTIONS B01-08)

Abstract. Practice problems for the eighth lecture of Part II, delivered May 17 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Consider the unique plane π containing the three points $P = (1, 0, 2), Q = (-2, 3, 0)$ and $R = (0, -5, 1)$.

(a) Find the distance from the point $S = (1, 2, -4)$ to π using the vector $\vec{PS}$.
(b) Find the distance from the point $S = (1, 2, -4)$ to π using the vector $\vec{QS}$.
(c) Find the distance from the point $S = (1, 2, -4)$ to π using the vector $\vec{RS}$.

Problem 2. Consider the three planes

$$\pi_1 = \{3x - 5y + 4z = 12\}$$

$$\pi_2 = \{\text{unique plane that contains } (0, 1, 0) \text{ with perpendicular direction } \langle 1, 4, 3\rangle\}$$

$$\pi_3 = \{\text{unique plane that contains } (0, 0, 0) \text{ and vectors } u = \langle 2, 4, 1\rangle, v = \langle 2, -5, 12\rangle\}$$

and the point $S = (-2, 0, 1)$.

(a) Find the distance of S to π_1.

(b) Find the distance of S to π_2.

(c) Find the distance of S to π_3.

Problem 3. Find two different points S_1 and S_2 in space such that both S_1 and S_2 have distance to the plane $\{x + y + z = 0\}$ equal to 9.

Problem 4. Find two different planes π_1 and π_2 in space such that both π_1 and π_2 have distance to the point $S = (1, 0, 0)$ equal to 23.

Problem 5. Consider the two planes

$$\pi_1 = \{x - z = 12\}$$

$$\pi_2 = \{\text{unique plane that contains } (0, 0, 0) \text{ with perpendicular direction } \langle 1, 1, 1\rangle\}$$

(a) Compute the distance from S to the plane π_1.
(b) Compute the distance from S to the plane π_2.
(c) Compute the distance from S to the intersection line $\pi_1 \cap \pi_2$.

Problem 6. Let L be the unique line through the point $P = (1, 2, 0)$ and direction vector $v = \langle 0, 2, -7 \rangle$. Compute the distance from the point $S = (-3, 0, 4)$ to the line L.

Problem 7. Let L be the unique line through the points $P = (1, 2, 0)$ and $Q = (7, -5, 6)$. Compute the distance from the point $S = (-3, 0, 4)$ to the line L.

Problem 8. Decide whether each of the following sentences is *true* or *false*.

(a) A point P belongs to a line L if and only if the distance from P to L is zero.

(b) A point P belongs to a plane π if and only if the distance from P to π is zero.

(c) Given a point P, there exists a unique plane π whose distance to P is 1.

(d) Given a point P, there are infinitely many lines L whose distance to P is 14.

(e) If a point P belongs to a plane π_1 and L is a line of intersection between π_1 and a different (non-parallel) plane π_2. Then the distance from P to L is the same as the distance from P to π_2.