MAT 21C: PRACTICE PROBLEMS LECTURE 9

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the ninth lecture of Part II, delivered May 19 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Find the parametric description of the trajectory of a particle which moves along the unique line through the point $P = (3, -5, 11)$ and with direction vector $\langle 7, 2, -1\rangle$.

Problem 2. Find the parametric description of the trajectory of a particle which moves along the unique line given by the intersection of the two planes $\pi_1 = \{x+4y-6z = 10\}$ and $\pi_2 = \{11x - 2y - 6z = 3\}$.

Problem 3. Suppose that a particle is moving along a line L with its trajectory being given by $r(t) = \langle 3 + 8t, 5 - 9t, 1 + t\rangle$.

(a) Find the position of the particle at $t = 0$ and $t = 5$.
(b) Find the velocity of the particle at $t = 0$ and $t = 5$.
(c) Give a point in the line L and a vector in the direction of the line L.

Problem 4. Consider a particle in space moving according to the trajectory

$$r(t) = \langle t^2 - 3t, 3 - t, 45t + e^t\rangle.$$

(a) What is the position of the particle at $t = 0$? And at $t = 10$?
(b) Compute the velocity vector of the particle at $t = 0$ and at $t = 10$.
(c) Find the speed of the particle at $t = 0$ and at $t = 10$.

Problem 5. A particle in space moves according to the trajectory

$$r(t) = \langle \cos(t^3), \sin(t^3), 0\rangle.$$

(a) Show that at $t = 0$ and at $t = \sqrt[3]{2\pi}$ the particle passes through the same point.

(b) Compute the speed at which is passes at $t = 0$ and at $t = \sqrt[3]{2\pi}$.

Problem 6. Given a particle moving according to the trajectory

$$r(t) = \langle \cos(t), \sin(t), 5e^{-t} \rangle.$$

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(a) Find the position of the particle at $t = 0$ and $t = \pi$.
(b) Find the velocity of the particle at $t = 0$ and $t = 5$.
(c) Find the acceleration of the particle at $t = 0$ and $t = 5$.
(d) Compute the angle of the velocity and the acceleration at $t = 0$ and $t = 5$.

Problem 7. Two particles move in space according to the trajectories

$$r_1(t) = \langle \cos(t), \sin(t), t \rangle, \qquad r_1(t) = \langle t, 2t, t \rangle.$$

Show that the two particles will never collide.

Problem 8. Two particles move in space according to the trajectories

$$r_1(t) = \langle \cos(t), \sin(t), t \rangle, \qquad r_1(t) = \langle 1, 0, t \rangle.$$

Show that the two particles will collide infinitely many times and find all such times of collision.

Problem 9. Suppose that a particle travels along a line L through the point $P = (p_1, p_2, p_3)$ and vector direction $v = \langle v_1, v_2, v_3 \rangle$ via the parametric trajectory

$$r(t) = \langle p_1 + tv_1, p_2 + tv_2, p_3 + tv_3 \rangle.$$

(a) Show that the velocity vector of $r(t)$ is exactly the direction vector v of L.
(b) Show that the acceleration vector of $r(t)$ is always zero.

Problem 10. Find a parametric description of the curve given by $r(t) = \langle 1, 3t + 2, t \rangle$ where the particle moves exactly in the same trajectory but at twice the speed.