University of California Davis **Name (Print):** ____________________
Calculus MAT 21C **Student ID (Print):** ____________________

Midterm Examination **May 26 2023**
Time Limit: 50 Minutes

This examination document contains 11 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The maximum grade for this exam is **100 points**. Any grade surpassing that mark will still count for 100 points, which is the top grade. You can obtain 100 points in any combination of the problems below, the **extra 20 bonus points** can only help you. You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by no calculations, explanations, or algebraic work will receive less credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	30	
2	40	
3	30	
4	20	
Total:	120	

Do not write in the table to the right.

1. (30 points) Consider the three points $P = (1, 0, 2)$, $Q = (6, 5, -1)$ and $R = (-3, 0, -2)$.

   (a) (5 points) Find the vectors $\vec{PQ}$, $\vec{QR}$ and $\vec{PQ} + \vec{QR}$.

   (b) (5 points) Compute the length $|\vec{PQ}|$ and the midpoint between P and Q.

(c) (10 points) Find an equation for the unique plane π_1 containing P, Q and R.

(d) (5 points) Compute the distance from the point $S = (1, 0, 0)$ to the plane π_1.

(e) (5 points) Find the distance from the point $S = (1, 0, 0)$ to the line of intersection $L = \pi_1 \cap \pi_2$ of the two planes π_1 and $\pi_2 = \{z = 0\}$.

2. (40 points) Consider the two vectors $u = \langle 2, 5, -1 \rangle$ and $v = \langle 3, -4, 7 \rangle$.

   (a) (5 points) Compute the dot product $u \cdot v$.

   (b) (5 points) Find $\cos\theta$, where θ is the angle between the vectors u and v.

(c) (10 points) Compute the cross product $u \times v$.

(d) (5 points) Find the area of the parallelogram formed by u and v.

(e) (5 points) Is $\langle 4, 10, -2 \rangle$ parallel, perpendicular or neither to the vector u?

3. (30 points) Consider a particle moving according to the trajectory

$$r(t) = \langle \cos(2t), \sin(5t), t + e^{-t} \rangle.$$

(a) (5 points) Argue that the velocity at time $t = \pi$ is $v(\pi) = \langle 0, -5, 1 - e^{-\pi} \rangle$.

(b) (5 points) Compute the speed of the particle at time $t = \pi$.

(c) (5 points) Is the position $r(\pi)$ at time $t = \pi$ perpendicular to the velocity $v(\pi)$?

(d) (10 points) Find the acceleration vector $a(t)$.

(e) (5 points) Consider the sphere $\{x^2 + y^2 + z^2 = 4\}$. Explain why at time $t = 0$, the position $r(0)$ of the particle is *inside* the sphere.

4. (20 points) For each of the five sentences below, circle the correct answer. There is a unique correct answer per item. (You do *not* need to justify your answer.)

   (a) (5 points) The intersection of the ball $\{(x-4)^2+(y+5)^2+z^2 \leq 36\}$ with the plane $\pi = \{x+y+z=-1\}$ is:

   (1) Empty. (2) A circle. (3) A disk. (4) A half-space. (5) A line.

   (b) (5 points) The cross product $u \times v$ of $u = \langle 1,2,-1\rangle$ and $v = \langle -3,-6,3\rangle$ is:

   (1) $\langle 3,-6,-3\rangle$ (2) $\langle 0,1,0\rangle$ (3) $\langle 2,4,-2\rangle$ (4) $\langle 0,-1,0\rangle$ (5) $\langle 0,0,0\rangle$.

   (c) (5 points) The distance from $S = (1,3,0)$ to the plane $\pi = \{2x+y-z=5\}$ is:

   (1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2 (5) $\sqrt{3}$.

   (d) (5 points) The dot product $u \cdot v$ of $u = \langle 3,0,-4\rangle$ and $v = \langle 5,2,1\rangle$ is:

   (1) 0 (2) 3 (3) 7 (4) 11 (5) 15.

   (e) (5 points) A particle has trajectory $r(t) = \langle t, e^t, \cos(t)\rangle$. Its initial speed at $t=0$ is:

   (1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2 (5) $\sqrt{3}$.