**University of California Davis** **Name (Print):** ____________________

**Calculus MAT 21C** **Student ID (Print):** ____________________

**Practice Final Examination** **June 12 2023**

**Time Limit: 120 Minutes**

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This examination document contains 12 pages, including this cover page, and 5 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The maximum grade for this exam is **100 points**. Any grade surpassing that mark will still count for 100 points, which is the top grade. You can obtain 100 points in any combination of the problems below, the **extra 20 bonus points** can only help you. You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by no calculations, explanations, or algebraic work will receive less credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use back of the pages; clearly indicate when you have done this.

| Problem | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| Total: | 100 | |

Do not write in the table to the right.

1. (20 points) Consider the function $f(x) = \ln(1+x^2)\cos(4x)$.

   (a) (4 points) Compute the Taylor series of $\cos(4x)$ centered at $a = 0$.

   (b) (4 points) Justify why the Taylor series of $\ln(1+x^2)$ centered at $a = 0$ is

$$\ln(1+x^2) \approx x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \ldots = \sum_{n\geq 1}(-1)^{n+1}\frac{x^{2n}}{n}.$$

(c) (4 points) Find the radius of converge of the Taylor series in Part (b).

(d) (4 points) Argue that the Taylor approximation of order 7 of $f(x)$ at $a = 0$ is

$$f(x) \approx x^2 - \frac{17x^4}{2} + 15x^6.$$

(e) (4 points) The error of the Taylor approximation of order 7 of $f(x)$ centered at $a = 0$ in the interval $x \in [0, 0.1]$ is bounded by $10^{-6}$. Using the Taylor approximation in Part (d), find the numerical value of

$$f(0.1) = \ln(1 + 0.01)\cos(0.4)$$

with an accuracy of 5 decimal digits, i.e. at least 5 decimal digits correct.

2. (20 points) Consider the three points $P = (2, 1, 0)$, $Q = (3, 0, 4)$ and $R = (-5, 1, -1)$.

   (a) (4 points) Explain why the vectors $u = \vec{PQ}$ and $v = \vec{PR}$ are

$$u = \langle 1, -1, 4\rangle, \quad v = \langle -7, 0, -1\rangle.$$

   (b) (4 points) Compute $\cos\theta$, where $\theta$ is the angle between $u$ and $v$.

(c) (4 points) Calculate the cross product $u \times v$ and find the area of the parallelogram spanned by $u$ and $v$.

(d) (4 points) Find an equation for the plane $\pi$ that contains the points $P, Q, R$.

(e) (4 points) What is the distance from the point $S = (1, 0, 1)$ to the plane $\pi$?

3. (20 points) Consider a particle moving according to the trajectory

$$r(t) = \langle e^{-t}\sin(t), e^{-3t}\cos(t), t^5 + t + 3\rangle.$$

(a) (5 points) Find the velocity $v(t)$ of the particle and its acceleration $a(t)$.

(b) (5 points) What is the speed of the particle at $t = 0$?

(c) (5 points) Consider the sphere $S = \{x^2 + (y-1)^2 + (z-3)^2 = 25\}$. Decide whether the particle is *inside*, *on* or *outside* of the sphere $S$ at the times $t = 0$ and $t = 10$.

(d) (5 points) Compute the distance from the initial position $r(0)$ of the particle to the line given by the intersection of the two planes $\{x = 0\}$ and $\{y = 2\}$.

4. (20 points) Consider the function

$$f(x,y) = x^2y^2 - x^2 - y^2 + 5.$$

(a) (5 points) Compute the partial derivatives $\partial_x f$ and $\partial_y f$.

(b) (5 points) Find all the critical points of $f(x,y)$.

(c) (5 points) Compute the second partial derivatives $\partial_{xx} f$, $\partial_{yy} f$, $\partial_{xy} f$, and $\partial_{yx} f$.

(d) (5 points) Classify all the critical points of $f(x, y)$ into minima, saddles or maxima.

5. (20 points) For each of the five sentences below, circle the correct answer. There is a unique correct answer per item. (You do *not* need to justify your answer.)

   (a) (4 points) The series $\sum_{n=1}^{\infty} \frac{n^2 - 3n + 4}{n^\alpha + n - 5}$ converges

   (1) if $\alpha > 3$. (2) if $\alpha \geq 3$. (3) if $\alpha \leq 3$. (4) converges for all $\alpha$.

   (b) (4 points) The value of the infinite series $\sum_{n=0}^{\infty} \frac{5}{7^n}$ is

   (1) 7/6 (2) 6/7 (3) 35/6 (4) 5/7 (5) 5/6 (6) $\infty$

   (c) (4 points) The intersection of $\{x+2y-3z = 10\}$ with $\{(x-4)^2+(y-3)^2+z^2 \leq 10\}$ is

   (1) a circle (2) a disk (3) a plane minus a disk (4) two points (5) empty

   (d) (4 points) The midpoint between $P = (-2, 3, -1)$ and $Q = (5, 4, -6)$ is

   (1) $(3, 7, -7)$ (2) $(3/2, 7/2, -7/2)$ (3) $(-7, -1, 5)$ (4) $(7, 1, -5)$ (5) $(7/2, 1/2, -5/2)$

   (e) (4 points) The radius of convergence of the Taylor expansion of $\ln(1+x) - \ln(1-x)$ is

   (1) 0 (2) 1/2 (3) 1 (4) 2/3 (5) $\pi/2$ (6) $\infty$