University of California Davis | **Name (Print):** ____________
Calculus MAT 21C | **Student ID (Print):** ____________

Practice Midterm II | **April 28 2023**
Time Limit: 50 Minutes

This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

Do not write in the table to the right.

1. (25 points) Consider the sequence $a_n = \dfrac{8^n}{n!}$ for $n \geq 1$.

   (a) (5 points) Write the first 5 terms of the sequence.

   (b) (5 points) Justify that the sequence (a_n) is neither increasing nor decreasing.

(c) (10 points) Argue that (a_n) is a convergent sequence.

(d) (5 points) Show that $\lim_{n\to\infty} a_n = 0$.

2. (25 points) Let us consider the series

$$S = \sum_{n=1}^{\infty} (-1)^n \cdot \left(\sqrt[n]{3} - 1 \right).$$

(a) (5 points) Show that the sequence $b_n = \sqrt[n]{3} - 1$, $n \geq 1$, converges to 0.

(b) (10 points) Argue that $b_{n+1} \leq b_n$ for $n \in \mathbb{N}$ large enough.

(c) (5 points) Explain why the series S is convergent.

(d) (5 points) Does the following series converge?

$$\sum_{n=1}^{\infty}(-1)^n \cdot \sqrt[n]{3}.$$

3. (25 points) The goal of this problem is to find the value sin(0.2) with an error less than 10^{-6}, i.e. so that the first 6 decimal digit are accurate.

   (a) (5 points) Explain why the Taylor expansion of $\sin(x)$ at $a = 0$ is

$$\sin(x) \overset{(x\approx 0)}{=} x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

   (b) (10 points) Show that the Taylor truncation of degree 5 at $a = 0$ has error bounded by 10^{-6} at $x = 0.2$.

(c) (5 points) Evaluate $\sin(0.2)$ with the first 5 decimal digits being exactly accurate.

(d) (5 points) Please find the mistake in the following (wrong) argument: *the error in the Taylor series trucated at degree* 5 *is given by the next term in the Taylor series. Therefore, the error is given by*

$$\frac{f^{(6)}(0)}{6!}x^6 = \frac{sin^{(6)}(0)}{6!}x^6 = \frac{-sin(0)}{6!} = 0.$$

In conclusion, the Taylor truncation of degree 5 *actually approximates* $\sin(x)$ *with no error. Thus* $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ *exactly.*

4. (25 points) For each of the ten sentences below, circle the correct answer. (You do *not* need to justify your answer.)

   (a) (5 points) The value of the infinite series $\sum_{n=0}^{\infty}(-1)^n \cdot \left(\frac{3}{2^n}\right)$ is

   (1) 0. (2) 1 (3) 2 (4) 3 (5) ∞.

   (b) (5 points) The Taylor expansion of $\ln(1+x^2)$ at $x=0$ of order 6 is

   (1) $x - \frac{x^2}{2} + \frac{x^3}{3}$. (2) $x + \frac{x^2}{2} + \frac{x^3}{3}$ (3) $x^2 - \frac{x^4}{2} + \frac{x^6}{3}$ (4) $x^2 - \frac{x^4}{4} + \frac{x^6}{6}$.

   (c) (5 points) The ratio test applied the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$

   (1) concludes convergence. (2) concludes divergence (3) is inconclusive

   (d) (5 points) The value of the geometric series $\sum_{n=0}^{\infty} 5^n$ is

   (1) 0 (2) -0.25 (3) 0.25 (4) 0.5 (5) ∞

   (e) (5 points) If a sequence (a_n) converges then (a_n) is bounded below.

   (1) True. (2) False.