University of California Davis
Calculus MAT 21C

Name (Print): ____________________
Student ID (Print): ____________________

Practice Midterm Examination
Time Limit: 50 Minutes

May 26 2023

This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Consider the points $P = (0, 3, -2)$ and $Q = (4, -3, -11)$.

   (a) (5 points) Find the vector $\vec{PQ}$.

   (b) (5 points) Show that $|\vec{PQ}| = \sqrt{133}$, i.e. the length equals the square root of 133.

(c) (10 points) Find the midpoint of P and Q.

(d) (5 points) Consider the sphere given by the equation

$$x^2 + (y-3)^2 + (z+2)^2 = 4.$$

For each of the two points P and Q, decide whether they are *inside* the sphere, *on* the sphere or *outside* the sphere.

2. (25 points) Consider the vectors $u = \langle 3, 1, -4 \rangle$ and $v = \langle 2, -10, 5 \rangle$.

   (a) (5 points) Compute the lengths $|u|$ and $|v|$.

   (b) (5 points) Find the dot product $u \cdot v$.

   (c) (5 points) Show that the angle θ between u and v satisfies

$$\cos\theta = -4 \cdot \sqrt{\frac{6}{559}}.$$

(d) (5 points) Find the cross product $u \times v$.

(e) (5 points) Find the area of the parallelogram spanned by u and v.

3. (25 points) Consider the point $P = (5, 0, 13)$ and the plane π given by the equation

$$\pi := \{x + 2y - z = 10\}.$$

(a) (5 points) Find a direction vector in the perpendicular direction of π.

(b) (5 points) Compute the distance from P to π.

(c) (10 points) Find the unique plane π' which contains the point P and the two points $Q = (0, 0, 0)$ and $R = (1, 0, 0)$.

(d) (5 points) Find the distance from the point P to the line $L = \pi \cap \pi'$ given by the intersection of π and π'.

4. (25 points) For each of the statements below, circle the **unique** correct answer. (You do *not* need to justify your answer.)

   (a) (5 points) The intersection of the sphere $x^2 + y^2 + (z-3)^2 = 1$ with the plane $\pi = \{x - y + z = 3\}$ is:

   (1) Empty. (2) A circle. (3) A disk. (4) A half-space. (5) A line.

   (b) (5 points) A particle moves in space following the trajectory $r(t) = \langle \cos(t), t^2, \sin(t) \rangle$. Its speed at $t = 0$ is:

   (1) 0 (2) 0.5 (3) 1 (4) 1.5 (5) 2.

   (c) (5 points) The distance from $P = (1, 3, -16)$ to the plane $\pi = \{3x+2y+z = -7\}$ is:

   (1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2 (5) $\sqrt{3}$.

   (d) (5 points) Given the trajectory $r(t) = \langle 15t + 4, -t, \cos(2t) \rangle$, the acceleration vector at time $t = 2\pi$ is given by the vector:

   (1) $\langle 15, -1, 0 \rangle$ (2) $\langle 0, 1, 0 \rangle$ (3) $\langle 0, 0, -4 \rangle$ (4) $\langle 0, -1, 2 \rangle$ (5) $\langle 15, -1, 2 \rangle$.

   (e) (5 points) A particle moves with trajectory $r(t) = (t, 6t, 3t)$ along a line. In its entire trajectory (for all t), it intersects the sphere $\{x^2 + y^2 + z^2 = 16\}$:

   (1) Never. (2) At 1 point. (3) At 2 points. (4) At 3 points.