This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. Fill in all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may not use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this and explain why the theorem may be applied.

(B) Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.
1. (25 points) Consider the points $P = (0, 3, -2)$ and $Q = (4, -3, -11)$.
   
   (a) (5 points) Find the vector $\vec{PQ}$.

   (b) (5 points) Show that $|\vec{PQ}| = \sqrt{133}$, i.e. the length equals the square root of 133.
(c) (10 points) Find the midpoint of $P$ and $Q$.

(d) (5 points) Consider the sphere given by the equation

$$x^2 + (y - 3)^2 + (z + 2)^2 = 4.$$ 

For each of the two points $P$ and $Q$, decide whether they are inside the sphere, on the sphere or outside the sphere.
2. (25 points) Consider the vectors \( u = \langle 3, 1, -4 \rangle \) and \( v = \langle 2, -10, 5 \rangle \).

(a) (5 points) Compute the lengths \(|u|\) and \(|v|\).

(b) (5 points) Find the dot product \( u \cdot v \).

(c) (5 points) Show that the angle \( \theta \) between \( u \) and \( v \) satisfies

\[
\cos \theta = -4 \cdot \sqrt{\frac{6}{559}}.
\]
(d) (5 points) Find the cross product \( u \times v \).

(e) (5 points) Find the area of the parallelogram spanned by \( u \) and \( v \).
3. (25 points) Consider the point \( P = (5, 0, 13) \) and the plane \( \pi \) given by the equation

\[
\pi := \{ x + 2y - z = 10 \}.
\]

(a) (5 points) Find a direction vector in the perpendicular direction of \( \pi \).

(b) (5 points) Compute the distance from \( P \) to \( \pi \).
(c) (10 points) Find the unique plane $\pi'$ which contains the point $P$ and the two points $Q = (0, 0, 0)$ and $R = (1, 0, 0)$.

(d) (5 points) Find the distance from the point $P$ to the line $L = \pi \cap \pi'$ given by the intersection of $\pi$ and $\pi'$. 
4. (25 points) For each of the statements below, circle the unique correct answer. (You do not need to justify your answer.)

(a) (5 points) The intersection of the sphere \( x^2 + y^2 + (z - 3)^2 = 1 \) with the plane \( \pi = \{x - y + z = 3\} \) is:


(b) (5 points) A particle moves in space following the trajectory \( r(t) = \langle \cos(t), t^2, \sin(t) \rangle \). Its speed at \( t = 0 \) is:

(1) 0 (2) 0.5 (3) 1 (4) 1.5 (5) 2.

(c) (5 points) The distance from \( P = (1, 3, -16) \) to the plane \( \pi = \{3x + 2y + z = -7\} \) is:

(1) 0 (2) 1 (3) \( \sqrt{2} \) (4) 2 (5) \( \sqrt{3} \).

(d) (5 points) Given the trajectory \( r(t) = \langle 15t + 4, -t, \cos(2t) \rangle \), the acceleration vector at time \( t = 2\pi \) is given by the vector:

(1) \( \langle 15, -1, 0 \rangle \) (2) \( \langle 0, 1, 0 \rangle \) (3) \( \langle 0, 0, -4 \rangle \) (4) \( \langle 0, -1, 2 \rangle \) (5) \( \langle 15, -1, 2 \rangle \).

(e) (5 points) A particle moves with trajectory \( r(t) = (t, 6t, 3t) \) along a line. In its entire trajectory (for all \( t \)), it intersects the sphere \( \{x^2 + y^2 + z^2 = 16\} \):

(1) Never. (2) At 1 point. (3) At 2 points. (4) At 3 points.