This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. Fill in all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may not use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this and explain why the theorem may be applied.

(B) Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.
1. (25 points) Consider the points $P = (1, 0, 0)$, $Q = (-2, 0, 3)$ and $R = (-5, 1, -1)$.
   
   (a) (5 points) Compute the vector $\vec{PQ} \times \vec{PR}$.
   
   (b) (5 points) Consider the unique plane $\pi$ containing $P$, $Q$ and $R$. Explain why
   
   \[ \{-3x - 21y - 3z = -3\} \]
   
   is an equation for $\pi$. 
(c) (5 points) Justify that \( v = (21, 0, -21) \) is a direction of the line \( L \) of intersection of \( \pi \) with the plane \( \Pi = \{x + z = 1\} \).

(d) (5 points) Find the distance from \( S = (0, 0, 2) \) to line \( L \).

(e) (5 points) Find the distance from \( S = (0, 0, 2) \) to the plane \( \pi \).
2. (25 points) Consider the vectors \( u = \langle 2, 0, -1 \rangle \) and \( v = \langle 3, 4, -5 \rangle \).

(a) (5 points) Show that \( \langle 4, 7, 8 \rangle \) is perpendicular to both \( u \) and \( v \).

(b) (5 points) Argue that \( u \) is not parallel to \( v \).
(c) (5 points) Compute $\sin \theta$, where $\theta$ is the angle between $u$ and $v$.

(d) (5 points) Verify that the vector $w = \langle 1, 0, 2 \rangle$ is perpendicular to $u$ but $w$ is not perpendicular to $v$.

(e) (5 points) Find a vector that is perpendicular to $v$ but not perpendicular to $u$. 
3. (25 points) Consider a particle moving with a trajectory \( \mathbf{r}(t) = (\cos(3t), \sin(4t), t^3) \).

(a) (5 points) Where will the particle be at time \( t = \pi \)?

(b) (5 points) Find the velocity vector \( \mathbf{v}(t) \) of the particle.

(c) (5 points) Compute the speed of the particle at time \( t = \pi \).
(d) (5 points) Show that the acceleration at $t = \pi$ is given by

$$a(\pi) = \langle 9, 0, 6\pi \rangle.$$

(e) (5 points) Will there ever be a positive time $t$ where the particle will be at rest, i.e. have zero speed?
4. (25 points) For each of the statements below, circle the unique correct answer. (You do not need to justify your answer.)

(a) (5 points) The intersection of the ball \((x - 2)^2 + y^2 + (z + 1)^2 \leq 16\) with the plane \(\pi = \{2x - 11y + 5z = -1\}\) is:


(b) (5 points) The intersection of the plane \(\pi_1 = \{x + y + z = 1\}\) with the plane \(\pi_2 = \{5x + 5y + 5z = 17\}\) is:

(1) Empty. (2) A circle. (3) A line. (4) A point. (5) Two points.

(c) (5 points) The cross product of \(u = \langle -3, 2, 4 \rangle\) and \(v = \langle 6, -4, -8 \rangle\):

(1) \(\langle 0, 0, 0 \rangle\) (2) \(\langle 1, 0, 0 \rangle\) (3) \(\langle 0, 1, 0 \rangle\) (4) \(\langle 0, 0, 1 \rangle\) (5) \(\langle 1, 1, 1 \rangle\).

(d) (5 points) The midpoint between \(P = (0, 6, 4)\) and \(Q = (8, 2, -4)\) is:

(1) \(\langle 4, 4, 0 \rangle\) (2) \(\langle 4, -2, -4 \rangle\) (3) \(\langle 8, -4, 8 \rangle\) (4) \(\langle 0, -8, 4 \rangle\) (5) \(\langle 2, 0, -4 \rangle\).

(e) (5 points) A particle with trajectory \(r(t) = (e^t, t + 3, 5t)\) has speed at \(t = 0\):

(1) 0. (2) \(\sqrt{25}\). (3) \(\sqrt{26}\). (4) \(\sqrt{27}\).