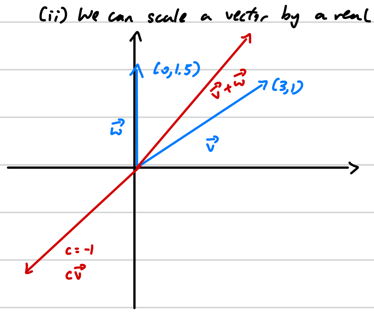
Subspace of vector spaces (4.3)

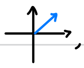
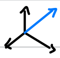
Recall: V is an $\mathbb{R}$ vector space — elements in V are called vectors

(i) we have a sum of vectors

(ii) we can scale a vector by a real scalar $c \in \mathbb{R}$



Examples

① $V = \mathbb{R}^n$, $n=2: \mathbb{R}^2$ , $n=3: \mathbb{R}^3$ 

② $V = \mathbb{R}[x] := \{$polynomials in x with $\mathbb{R}$ coeff$\}$

here $v \in V$ a vector is $v = 3 - x^2 + 5x^3 + x^7$ (can be rewritten as $\vec{v} = (3,0,-1,5,0,0,0,7)$ ← not canonical)

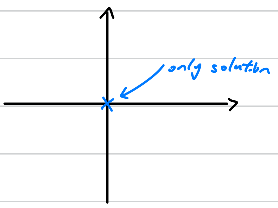
Solving systems of linear eq^n

Ex(1) $\left.\begin{array}{l} x_1 - x_2 = 0 \\ 3x_1 + 7x_2 = 0 \end{array}\right\}$ what are solution?

(x_1, x_2) unknowns we think as a vector $\rightsquigarrow$ $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in V = \mathbb{R}^2$

Solving,

$\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}\right\}$ only solution



Ex(2) $\left.\begin{array}{l} x_1 + x_2 = 0 \\ 3x_1 + 3x_2 = 0 \end{array}\right\}$ → homogeneous linear equations

solution are $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$

$x_1 := -x_2$ is the only constraints (i.e. $\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ or $\vec{w} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ are sol^n)

$\boxed{\text{Then } \vec{v} + \vec{w} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \text{, also a solution}}$

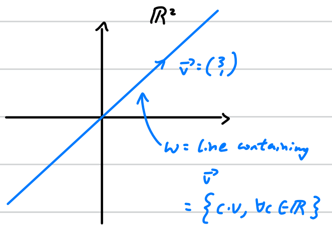
$\boxed{\text{Similarly, } c \cdot (\vec{v}) = \begin{pmatrix} c \\ -c \end{pmatrix} \text{ is also a solution } \forall c \in \mathbb{R}}$

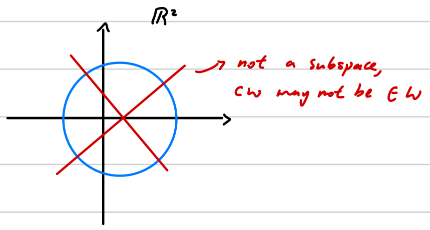
Def^n: A vector subspace of V is a subset $W \subseteq V$ such that sums & scalar multiplication of V when restricted to W make W into a vector space.

→ 2 vectors $w_1, w_2 \in W$ satisfy $w_1 + w_2 \in W$ (not just V)

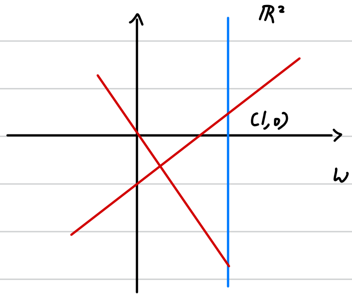
Examples ① $V = \mathbb{R}^n$

$\mathbb{R}^2 = V$





Observation: Vector subspaces must contain the origin



not a vector subspace:

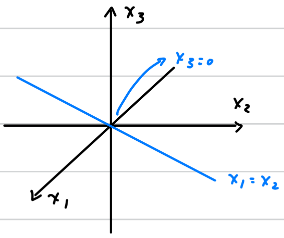
cw is not $\in W$,

$w_1 + w_2$ is not $\in W$

$W = \{(x_1, x_2) \text{ with } x_1 = 1\}$

Example $V = \mathbb{R}^3$

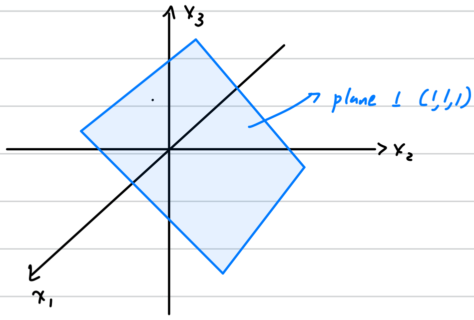
$W_1 = \{x_3 = 0,\ x_1 - x_2 = 0\}$



$W_2 = \{x_1 + x_2 + x_3 = 0\}$

ψ: $\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}, \begin{pmatrix}2\\3\\-5\end{pmatrix} \dots$

$\begin{pmatrix}\alpha\\-\alpha\\0\end{pmatrix}\ \begin{pmatrix}2\alpha\\3\alpha\\-5\alpha\end{pmatrix}$



Example: $V = \mathbb{R}[x]$, inside consider $W \subseteq V$ given by:

$W := \{p(x) \in \mathbb{R}[x] : p(0) = 0\}$ ⟵ $p(0) = a_0 + \dots + a_n 0^n = 0$

such that

i.e. $a_0 = 0$

$a_0 + a_1 x + \dots + a_n x^n$

∴ W is a vector subspace as a polynomial with zero constant term.

BUT ($W' = \{p \in \mathbb{R}[x] : p(0) = -3\}$ IS NOT A SUBSPACE$_{//}$

Theorem:

Let $V = \mathbb{R}^n$ and consider the linear system

$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} x_n = 0$ $\quad (a_{ij} \in \mathbb{R})$

j which x_j called (column)

i which eq^n (row)

$a_{21} \cdot x_1 + \dots + a_{2n} x_n = 0$

$\vdots$

$a_{n1} + \dots + a_{nn} x_n = 0$

Then $W = \left\{ \begin{pmatrix}x_1\\\vdots\\x_n\end{pmatrix} \in V : \begin{pmatrix}x_1\\\vdots\\x_n\end{pmatrix} \text{ solves the system} \right\} \subseteq V$ is A SUBSPACE